Math 331.2: Homework 13 (Section 7.6, 7.8, 9.1)

For the following systems

(a) Determine the type of the systems, *a saddle, source (also called stable node) or a sink (unstable node), improper node (degenerate eigenvalues), spiral sink, spiral source, center.*

(b) Sketch typical solution in in the phase plane (the x_1-x_2 plane). Indicate clearly the eigenvectors if adequate and sketch some typical solutions.

(c) Sketch a typical graph of x_1 vs. t

You **do not** need to write down the general solutions.

1. $\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \mathbf{x}$

2. $\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 2 & -1 \\ 6 & -2 \end{pmatrix} \mathbf{x}$

3. $\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} \mathbf{x}$

4. $\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix} \mathbf{x}$

5. $\frac{d\mathbf{x}}{dt} = \begin{pmatrix} -1 & -1 \\ 0 & -\frac{1}{4} \end{pmatrix} \mathbf{x}$

6. $\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \mathbf{x}$

For the following systems find the solution of the initial value problem. Sketch the graph of the two components $x_1(t)$ and $x_2(t)$ of the solutions $\mathbf{x}(t)$ and describe their behavior as $t \to \infty$.

7. $\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

8. $\frac{d\mathbf{x}}{dt} = \begin{pmatrix} -2 & -2 \\ 4 & 2 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

For the following systems

(a) Determine the eigenvalues in terms of α.

(b) Find the critical values of α where the type of the systems change.

(c) Draw a phase portrait of the system in the phase plane ($x_1 - x_2$-plane) for value of α just below the critical values of α and just above the critical values of α

9. $\frac{d\mathbf{x}}{dt} = \begin{pmatrix} \alpha & 1 \\ -1 & \alpha \end{pmatrix} \mathbf{x}$

10. $\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 2 & -5 \\ \alpha & -2 \end{pmatrix} \mathbf{x}$

12. $\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 0 & -5 \\ 1 & \alpha \end{pmatrix} \mathbf{x}$