Math 456: Homework 3

1. Math 478 has two section. In section I there is 12 female and 18 male students. In section II there are 20 female and 15 male students. The professor picks a section at random and then picks a student at random in that section. Compute

   (a) Probability that the student chosen is a female.

   (b) The conditional probability that the student is in section II given that she is a female

2. **(False positives, Sensitivity and Specificity).** This is standard use of Bayes formula. Suppose we deal with a disease and we have test for the disease. We know

   - The *sensitivity* of the test is 99%, this means that if you have the disease, the test is positive with probability 0.99.
   - The *specificity* of the test is 98%, this means that if you do not have the disease, the test is negative with probability 0.98.
   - The *prevalence* of the disease is one in two hundred, this means that the probability to carry the disease is 0.005.

   The probability that someone with a positive test actually has the disease is called the *positive predictive value* of the test. The probability that someone with a negative test actually does not have the disease is called the *negative predictive value* of the test.

   Express the positive and negative predictive value of the test using conditional probabilities and compute them using Bayes formula.

3. **(Piranha in a fishbowl)**.

   (a) In a (opaque) fishbowl there is a fish which is equally likely to be a piranha or a gold fish. A sushi lover throws a piranha into the fishbowl and immediately (before they can eat each other) removes one of the two fishes from the bowl and this turns out to be a Piranha. What is the probability that fish originally in the bowl was a piranha?

   (b) Repeat the same problem but now assuming that originally the probability that the fish was piranha was equal to p?

4. **(Monty's Hall problem)** In this problem you will analyze how important the assumptions are in Monty's Hall problem.

   (a) Assume as before that the prize is randomly put behind one the three doors. However after watching the game many times you notice that the show host who is usually standing near door 1 is lazy and he tends to open the door closest to him. For example if you pick door 1, he opens door 2 75% of the time and door 3 only 25% of the time. Does this modify your probability of winning if you switch (or not switch)?

(b) Suppose you have been watching Monty's Hall game for a very long time and have observed that the prize is behind door 1 45% of the time, behind door 2 40% of the time and behind door 3 15% of the time. The rest is as before and you assume that the host opens door at random (if he has a choice).

i. When playing on the show you pick door 1 again and the host opens one empty door. Should you switch? Compute the various cases.

ii. If you know you are going to be offered a switch would it be better to pick another door rather than door 1? Explain.

5. **(Bayes rule in odds form)** Prove the following very elegant formula which expresses how the odds of an event are changed when new information appears. We call the event H the "hypothesis" (whose probability we are interested in) and we call the event E the "evidence" (this is what we observe to happen).

$$\frac{P(H|E)}{P(\overline{H}|E)} = \frac{P(E|H)}{P(E|\overline{H})} \times \frac{P(H)}{P(\overline{H})}$$

$$\text{posterior odds} = \text{ likelihood ratio} \times \text{ prior odds}$$

In this context one may call A to be the hypothesis and B the evidence

6. Consider the following game: three cards are placed in a hat. One card is black on both sides, one card is red on both sides, and one card is red one side and black on the other side. Then one participant is asked to pick a card at random from the hat, keeping only one side visible. Then the owner of the hat bets the participant equal odds that the other side of the card will be the same color as the one shown. The question is whether this is a fair bet or not?

(a) Without loss of generality you may assume that the visible side of the card is red. To use Bayes rule in odds form form the previous problem determine what is the hypothesis H and what is the evidence E?

(b) Use the Bayes rule in odds form to determine if the bet fair?

7. Its is known that either individual X or individual Y is the perpratator of a murder. Further investigation shows that the murderer is blood type A and it is known that %10 percent of the population if of blood type A. It turns out that individual X has blood type A while the blood type of individual Y remains undetermined.

Based on this information and using Bayes formula assign a probability that individual X is the murderer.