

Math 456: Homework 2

1. In the powerball lottery 5 white balls are drawn out of a drum which 69 balls and one red ball is drawn out of another drum with 26 balls. The balls are drawn without replacement and the order in which balls are drawn does not matter. The prize of the ticket is \$2 and there are 9 ways to win given in the table below. Compute the corresponding probabilities.

Balls	Prize	Probabilities
5 white & 1 red	Jackpot	
5 white	\$1,000,000	
4 white & 1 red	\$50,000	
4 white	\$100	
3 white & 1 red	\$100	
3 white	\$7	
2 white & 1red	\$7	
1 white & 1red	\$4	
1 red	\$4	

2. A six card hand is dealt from an deck of 52 cards. Find the probabilities that
- (a) All six cards are hearts
 - (b) There are three aces, two kings and one queen.
 - (c) There three cards of one suit and three of another suit.
3. Compute the probabilities to obtain the following poker hands
- (a) Two pairs
 - (b) A straight flush: fives cards of the same suit in order (e.g. 6, 7, 8, 9, 10 of hearts). (recall that the Ace can play two roles here...)
 - (c) A flush : five cards of the same suit but not in order (e.g. 3, 5,6, queen,and king of spades).
 - (d) A straight: five cards in order (e.g. 5, 6, 7, 8, 9) but not all of the same suite (this would be a straight flush).
4. Explain with a "story proof" why the identity $\binom{2n}{n} = \sum_{j=0}^n \binom{n}{j}^2$ holds.

Hint: Think of a group of consisting of n boys and n girls. I do not want a proof by induction here.

5. A coin is tossed three times. What is the probability that exactly two heads are tossed given that
- The first outcome was a head.
 - The first outcome was a tail.
 - The first two outcomes were heads.
 - The first two outcomes were tails.
 - The first outcome was a head and the third was a head.
6. Imagine a game where a player is handed two cards. A lousy opponent reveals one of his two cards which turns out to be an ace.
- (a) His first card is an ace. What is the probability that he has two aces? That is compute $P(2 \text{ aces} \mid \text{first card is ace})$.
 - (b) One of his card is an ace. What is the probability that he has two aces? That is compute $P(2 \text{ aces} \mid \text{one card is an ace})$.
 - (c) One of his card is the ace of spade. What is the probability he has two aces? That is compute $P(2 \text{ aces} \mid \text{one card is the ace of spade})$.

Hint: It is harder than it looks

7. For blackjack, in the case of 1 deck, 6 decks and infinitely many decks compute the probability that the player's two cards give her (a) 16, (b) 20.
8. Insurance bet at blackjack: if the dealer upcard is an ace you may be offered the possibility to do a side bet that the dealer has a natural (that is an ace and a 10). This bet pays to 2 to 1 (that is you make a profit of \$2 if you bet one dollar and win). We assume here that we play with one deck and that you have looked at your two cards. In which situation should you take this bet?
9. Suppose your hand at blackjack is a total of 18 and the card up for the dealer is a 10. The dealer then reveals his card down which is a 3 and so he has a total of 13. Given this information and using the ∞ many decks assumption compute what are the probabilities that probability that
- (a) You win the game.
 - (b) You tie the game with the dealer.
 - (c) You lose the game.

Recall that the dealer hits repeatedly until he reaches 17 or higher and then stops.