

Math 456: Homework 1

1. What odds and what probabilities should you assign to the following events?

- (a) A card chosen at random from a 52-card deck is an ace.
- (b) Exactly two heads will turn up when three coins are tossed.

2. Bob and Maria are taking a math class with final grades A, B or C.

The probability that Bob gets a B is .3 and the probability that Maria gets a B is .4. The probability that neither gets an A but at least one gets a B is .1. What is the probability that at least one gets a B but neither gets a C?

Hint: This is not an easy problem. Write down the sample space S and consider the events $E = \{\text{Bob gets a B}\}$ and $F = \{\text{Maria gets a B}\}$. What are $E \cap B$ and $E \cup B$?

3. In a soccer tournament (with many teams) the odds that Arsenal wins are given as 2 to 7 while the odds that Liverpool wins are given as 1 to 5. Give the odds that either Arsenal or Liverpool wins?

4. The German Lotto consists of choosing 6 different numbers out of $1, 2, \dots, 49$ and thus there are $\binom{49}{6} = 13,986,816$ possible combinations. On June 21 in 1995, the drawing produced the numbers 15-25-27-30-42-48. Quite remarkably these were exactly the same numbers that on December 19 in 1986 and the first in the 3016 drawings of the German lotto that the same sequence had been drawn twice. What an amazing coincidence! Or is it really? Compute the probability that the same sequence of 6 numbers does not appear twice in 3016 drawings.

5. As seen in class if there are 23 people in a room the probability of having two people with the same birthday is more than $1/2$. In our class of 36 people what is this probability? For comparison compute the probability that somebody has the same birthday than you in our class? Are you surprised by the result?

6. In a horse race for which you are the bookmaker there are four horses and the amount of money which has been betted on each horse is

Euler	525
Bernoulli	900
Laplace	225
Fourier	350

- (a) Assign odds that for every horse to win the race based on that amount of money (the "true odds").

- (b) Now determine what are the fractional and decimal odds that you, as the bookmaker, you are going to assign to the four horses assuming you want to take %5 cut no matter which horse wins the race.
7. (a) Suppose you roll three dice repeatedly say m times. Compute the probability that he three dice will turn up sixes *at least once*.
- (b) How many times should you roll the three dice so that the probability you see three sixes at least once exceeds $\frac{1}{2}$?
8. The moneyline odds for the Superbowl LIII are, on January 24 2019, -132 for the New England Patriots and 116 for the Los Angeles Rams. In this problem we are going to figure out what is the commission taken by the bookmaker and then what are the "true odds" that the Patriots are going to win.
- (a) What are the corresponding fractional odds and decimal odds.
- (b) Alice happens to like gambling but at the same is very much risk averse (not a great combination to make big bucks but a great combination to figure out what is going on.....). She follow a so-called "minimax strategy". She decide to split her money (say \$ 100) and bet on both teams at the same time. She will bet $\$x$ on New England and $\$(100 - x)$ on Los Angeles. For a given x what is the *minimal* amount of money, $M(x)$, she will receive, no matter what happens.
- (c) Find the optimal choice x^* which will maximize $M(x)$. Comment on the choice x^* obtained in (c)? What does it say about the commission that the bookmaker takes on bets?
- (d) Deduce from this the "true" probability that the patriots are going to win (or rather the degree of collective belief that they are going to win!)