# DEPARTMENT OF MATHEMATICS AND STATISTICS <br> UNIVERSITY OF MASSACHUSETTS <br> MATH 131 Spring 2004 FINAL EXAM 

Your Section Number: $\qquad$

Your Instructor's Name: $\qquad$

Your ID Number: $\qquad$

Print Your Name: $\qquad$

Sign Your Name: $\qquad$

On this exam, you may use a calculator and a page of your own notes, but no books. It is not sufficient to just write the answers. You must show how you arrive at your answers unless instructed otherwise. If you draw a graph, show the numerical scale on each axis.

The entire exam is worth 140 points, with each problem worth 20 points.
Leave the space below empty!

1. (20) $\qquad$
2. (20) $\qquad$
3. (20) $\qquad$
4. (20) $\qquad$
5. (20) $\qquad$
6. (20) $\qquad$
7. (20) $\qquad$
8. (20 points) Please classify the following statements as True or False. Write out the word completely; do not simply write $T$ or $F$. There is no partial credit for this problem, and it is not necessary to show your work for this problem.
(a) (4 points) If one uses the tangent line approximation (i.e. linearization or differentials) to approximate $\sqrt{10}$, one obtains 19/6.
(b) (4 points) Suppose $f$ is a differentiable function such that $f(1)=3$ and $f^{\prime}(1)=2$. Then the equation of the tangent line to the graph of $f$ at $(1,3)$ is $y-2=3(x-1)$.
(c) (4 points) If $y=x^{x}$ then $y^{\prime}=x^{x} \ln x$.
(d) (4 points) If $f^{\prime \prime}(a)=0$, then $x=a$ is a point of inflection for the graph of $f(x)$.
(e) (4 points) Every continuous function with a closed and bounded domain has a global maximum and a global minimum value.
9. (20 points)
(a) (5 points) State the definition of the derivative as a limit.
(b) (15 points) Use the definition of the derivative to verify that the derivative of $f(x)=\sqrt{3 x+1}$ is $f^{\prime}(x)=\frac{3}{2 \sqrt{3 x+1}}$.
10. (20 points) Let $f(x)=\frac{x^{2}+1}{(x+1)^{2}}$. Then $f^{\prime}(x)=\frac{2 x-2}{(x+1)^{3}}$ and $f^{\prime \prime}(x)=\frac{8-4 x}{(x+1)^{4}}$.
(a) (2 points) What is the domain of $f$ ?
(b) (3 points) Where is the graph of $f$ increasing? Decreasing?
(c) (3 points) Where is the graph of $f$ concave up? Concave down?

This problem continues on the next page.
(d) (3 points) Does $f$ have any local maxima or minima, or points of inflection? If so, give the $x$-coordinates of these points.
(e) (3 points) Does $f$ have any horizontal or vertical asymptotes? If so, give their equations.
(f) (3 points) Use the information obtained in parts (a)-(e) to sketch the graph of $f$. Indicate clearly any features you discovered in (a)-(e).
4. (20 points) At noon, ship $A$ is 100 km west of ship $B$. Ship $A$ is sailing south at $35 \mathrm{~km} / \mathrm{hr}$, and ship $B$ is sailing north at $25 \mathrm{~km} / \mathrm{hr}$. How fast is the distance between the ships increasing at 4:00 PM?
5. (20 points) Compute the following limits. Recall that you must show your work to receive credit.
(a) (6 points) $\lim _{x \rightarrow \infty} \frac{\ln x}{x^{3}}$
(b) (7 points) $\lim _{x \rightarrow 0} \frac{x}{1-e^{-x}}$
(c) $(7$ points $) \lim _{x \rightarrow 0}(1+3 x)^{(2 / x)}$
6. (20 points) Find the global maximum and global minimum values of the function $f(x)=2 x^{3}+3 x^{2}+12 x-4$ on the interval $[-2,3]$. (Note: some instructors say absolute maximum and minimum instead of global, but it means the same thing.)
7. (20 points) Let $G$ be the portion of the graph of $y=48-x^{2}$ that lies on or above the $x$-axis. A rectangle is to be constructed with bottom corners on the $x$-axis and with top corners on $G$. Find the dimensions of such a rectangle with maximum area. Be sure to justify that your answer is indeed a maximum.

