

## Homework Problems

- 1) Consider the nonlinear dynamical lattice equation

$$\ddot{u}_n = C(u_{n+1} + u_{n-1} - 2u_n) - \sin(u_n)$$

Consider the limit of  $C = 0$  (so-called anti-continuum limit) and look for steady state solutions. Construct two kinds of kinks between two stable such steady states:

1. one that has half the particles in one steady state and the other half in the other (nothing in between). This is the so-called inter-site centered kink.
2. one that is as above but also has one site at the unstable steady state between them. This is a site-centered kink.

For  $C = 0$  the above are exact solutions. Now:

1. Perform a Newton iteration to find each of these solutions for finite  $C$  [using the previous step as your guess for the next one], from  $C = 0$  to  $C = 4$ .
2. Using the command `eig`, find numerically the eigenvalues of the Jacobian of the Newton. Plot the first two eigenvalues of the linearization around the intersite-centered and the first two around the site-centered kink, as a function of  $C$ . What is the continuum limit of these eigenvalues ?
3. Find the profile of the unstable kink at  $C = 0.5$ , and use it as initial condition for a 4th-order Runge-Kutta integrator that will evolve this solution as a function of time. Using the command `imagesc`, plot a space-time graph of the evolution. Can you find a way to plot the time evolution of the center of the solution vs. time ?
4. Attach the codes that you developed for the above (one code for the Newton and stability and one code for the time integration) and the requested plots.

- 2) Repeat the same steps as above but for the  $\phi^4$  model of the form:

$$\ddot{u}_n = C(u_{n+1} + u_{n-1} - 2u_n) + 2(u_n - u_n^3)$$

In this case, however, compute the lowest 3 eigenvalues of the linearization for the site-centered and inter-site centered kinks and identify their continuum limit.

### Practice Problems

1) Prove that for PDEs of the form  $u_{tt} = u_{xx} - f(u)$ , the spatial derivative  $\partial u_0 / \partial x$  is an eigenfunction with zero eigenvalue. Which symmetry does this allude to ?

2) Show that the linear momentum  $P = - \int u_t u_x dx$  is a conserved quantity for the above nonlinear Klein-Gordon dynamics  $u_{tt} = u_{xx} - f(u)$ .