Homework Problems

1) Consider the model:

$$\frac{dx_1}{dt} = (1 - x_1 - \frac{1}{2}x_2)\, x_1$$
$$\frac{dx_2}{dt} = (1 - \frac{2}{3}x_1 - x_2)\, x_2$$

(a) Find the equilibrium points in the quadrant $x_1, x_2 \geq 0$, and classify their types.

(b) Draw the phase portrait, with the nullclines. What is the predicted qualitative behavior of solutions in this model as time $t \to +\infty$?

Try to do part (b) by hand. Then go to the course webpage:

http://www.math.umass.edu/~kevrekid/math697

and download the code pplane7.m. Opening a Matlab window and executing pplane7 in the command window, use the program to produce the phase portrait that you will hand in (do make sure that it agrees with your predictions of the equilibrium points and of their types !).

2) Consider the standard diffusion equation $u_t = k u_{xx}$ in the interval $0 \leq x \leq l$ and for times $t \geq 0$, with $u(0,t) = u(l,t) = 0$. Solve the PDE using separation of variables $u(x,t) = X(x)T(t)$. Now, solve the corresponding lattice problem

$$u_j^{n+1} - u_j^n = \frac{k\Delta t}{\Delta x^2}\,(u_{j+1} + u_{j-1} - 2u_j)$$

with $u_0^n = u_J^n = 0$ also by separation of variables. Make sure that you get the same answer in the limit of $\Delta t \to 0$ and $\Delta x \to 0$ (and $J \to \infty$, so that $J\Delta x \to l$.

Practice Problems

1) Consider the initial value problem

$$\frac{dx}{dt} = -x \log x\,, \qquad x(0) = x_0 > 0\,,$$

which is a simplified and normalized model of tumor growth.

(a) Determine the solutions of this initial value problem, giving them in an explicit analytic form.

(b) Find the stable equilibrium point $x^* > 0$, show that these solutions approach x^* as $t \to +\infty$, and that in fact

$$x(t) - x^* \approx (\log x_0) \cdot e^{-t} \qquad \text{for large } t$$

2) Solve the wave equation $u_{tt} = c^2 u_{xx}$ with separation of variables $u(x,t) = X(x)T(t)$ and Dirichlet boundary conditions $u(0,t) = u(l,t) = 0$.