

Tentative Project Suggestions for Math 597/697

1. The nonlinear lattice version of the 1d Bose-Einstein condensate problem with a parabolic potential: For the linear problem, see: J. Phys. A: Math. Gen. **19**, L385 (1986) and for the nonlinear but continuum problem, see: J. Phys. B: At. Mol. Opt Phys. **38**, 1173 (2005).
2. The discrete nonlinear Schrödinger equation with long-range interactions, see e.g. Phys. Rev. E **62**, R53 (2000), Phys. Rev. E **61**, 4454 (2000), Phys. Rev. E **55**, 6141 (1997) and other papers by the same authors, as well as the case considered in pages 45-46 of the notes.
3. Solitary waves in the discrete Korteweg-de Vries equation, see e.g., J. Phys. Soc. Japan **60**, 2095 (1991) or Discrete and Continuous Dynamical Systems, 2005 Supplement Volume, pp. 22-29.
4. Traveling Waves in the discrete sine-Gordon equation: see e.g. the pioneering work of Physica D **14**, 88 (1984) and the recent developments of Physica D **237**, 50 (2008).
5. Traveling fronts and propagation failure in neural network / cardiac tissue models, see e.g., Physica D **136**, 1 (2000), SIAM J. Appl. Math **61**, 317 (2000), or Physica D **155**, 83 (2001).
6. Solitary waves in granular crystals, see e.g. Phys. Rev. E **56**, 6104 (1997), and more recent modeling developments such as e.g. Phys. Rev. Lett. **98**, 164301 (2007).
7. Solitons and breathers in dusty plasmas and Debye crystals, see e.g., Phys. Rev. E **76**, 016402 (2007) and references therein.
8. The role of defects and impurities in the statics and dynamics of solitary waves in nonlinear lattices, see e.g., the recent Phys. Rev. E **77**, 036614 (2008) and references therein.
9. Moving potentials and the generation of localized excitations in discrete lattices, see e.g. the continuum analog of the problem in Phys. Rev. E **55**, 2835 (1997).

10. Multi-component discrete systems, see e.g. the experimental work of Phys. Rev. Lett. **91**, 143907 (2003) and the theoretical analysis of Phys. Rev. E **73**, 066601 (2006).
11. Modulational instabilities in nonlinear lattices, see e.g., the original works of Opt. Lett. **13**, 794 (1988), Phys. Rev. A **46**, 3198 (1992) and the more recent experimental realization of Phys. Rev. Lett. **92**, 163902 (2004).
12. Numerous additional options are available including e.g. considering nonlinear lattices as a map problem (see p. 230 onward in the notes), considering multi-dimensional lattice problems and discrete vortices (see chapters III and IV in the notes), examining interactions of solitary waves in lattices or considering lattices with only few sites (again, see relevant sections in chapter VIII of the notes) as well as of course selecting a project of your own choice in consultation with PK.

What is Expected In the Project

In the project, you are expected to study *deeply* with analytical and/or numerical methods that you should find in the existing literature, a topic of your own choice relating to the application of a preferably nonlinear dynamical lattice model (or perhaps PDE), possibly stemming from some application in physics, chemistry, biology or engineering.

You should spend a considerable amount of time familiarizing yourself with the relevant model and examining in detail the methods developed mathematically or computationally to study it. You should be able to reproduce relevant calculations and computations and be familiar enough with them to present them in detail.

You are expected to compose a writeup of 10-20 pages which is to be sent to PK in .pdf form by the day of the class workshop. You are also expected on the day of the class workshop to make a 15-minute presentation of the work you carried out in your project over the semester.

A rough guideline for your writeup is the following.

- You should have an introduction presenting the physical problem and the model dynamical lattice or PDE that is relevant for it.
- You should have a results section that presents the analysis and mathematical and/or numerical results that you have obtained from it.
- Finally, you should also have a conclusions and future plans sections, detailing the outcome and impact of your results to the physical problem of interest and what possibly could be done in the future.

A rough timeline for the work of your project is as follows:

- In february, you should formulate the project in consultation with PK.
- In March you should read the relevant literature and finalize your plans about what to do (**by March 14, you should send to PK a title and abstract of what you plan to study and accomplish in your project**).
- In the rest of March, April and beginning of May you should spend a fair amount of time performing the relevant tasks.
- In May, you should also collect your results in the relevant writeup and presentation for the class workshop (that will take place right before the finals).