1. [10 points] The graphs of the functions $g$ and $f$ are pictured below:


$$
\int_{1}^{6} f(x) d x=3, \quad \int_{6}^{11} f(x) d x=-7, \quad \text { and } \int_{1}^{11} g(x) d x=10
$$

The area bounded by f and g is 30 .
a. Find $\int_{1}^{6} g(x) d x \quad$ and $\int_{6}^{11} g(x) d x$
b. Find the area bounded by $f(x)$ and the $x$ axis.
2. [9 points] Find the area of the region bounded $y=-x^{2}+8 x-11$ and $y=2 x-6$.
3. [9 points] Find $k$ so that

$$
1=\int_{0}^{\infty} \mathrm{ke}^{-4 t} \mathrm{dt}
$$

4. [9 points] Let C be the curve described by $\mathrm{x}(\mathrm{t})=2+2 \cos \mathrm{t}$ and $\mathrm{y}(\mathrm{t})=4+4 \sin ^{2} \mathrm{t}$ for $0 \leq \mathrm{t} \leq \pi$.
a. Find (dy/dx).
b. Find the x and y of the point where C has a horizontal tangent line.
c. Find an equation of the line tangent to C at $(3,7)$.
5. [16 points] Let $C$ be the curve described by $x(t)=1-2 \cos ^{2} t$ and $y=3 \sin ^{2} t$ for $0 \leq \mathrm{t} \leq \pi$.
a. Draw a graph of C with a window: $-2 \leq \mathrm{x} \leq 4$ and $-1 \leq \mathrm{y} \leq 4$.
b. Eliminate the parameter t . That is, find an equation that describes C with only variables x and y .
c. A particle travels along C from time 0 to time $\pi$. Find the distance that that particle travels, i.e. e., the arc length of $C$ from $t=0$ to $t=\pi$.
d. Find the distance from the initial point $(x(0), y(0))$ to the terminal point $((\mathrm{x}(\pi), \mathrm{y}(\pi))$, i.e. e. the displacement of the particle.
6. [9 points] Find the area inside of $r=2$ and to the right of $r=\sec \theta$.
7. [12 points] Find a power series that represents:
a. $\frac{2 \mathrm{x}}{1+\mathrm{x}^{2}}$
b. $\ln \left(1+x^{2}\right)$.
[Hint: $\sum_{n=0}^{\infty}(-1)^{n} x^{2 n}=\frac{1}{1+x^{2}}$ ]
8. [4 points ] State the Maclaurin series for
a. $\mathrm{e}^{\mathrm{x}}$
b. $\cos x$
9. [6 points] Let $\mathrm{F}(\mathrm{x})=\int_{0}^{\mathrm{x}} \sin \left(\mathrm{t}^{3}\right) \mathrm{dt}$. Find the Maclaurin series of $\frac{d(F(x))}{d x}$

$$
\left[\text { Hint: } \sin \mathrm{z}=\sum_{k=0}^{\infty}(-1)^{k} \frac{z^{2 k+1}}{(2 k+1)!}\right]
$$

10. [8 points] If $f^{(n)}(0)=\left(\frac{2}{3}\right)^{n}(n!)$
a. Find the Maclaurin series of $f(x)$
b. Find the radius of convergence.
11. [10 points]
a. Using the Taylor's polynomial (with $\mathrm{a}=0$ ) for the $\sin \mathrm{x}$ with a $5^{\text {th }}$ degree term compute an approximation of $\sin .2$.
b. Using Taylor's inequality find an upper bound for your answer in 11 a .
