Name (Last, First) ______________________________ **ID #** _____________

Signature ____

Lecturer ___________________________ **Section (A, B, C, etc.)** ________

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 132 **DRAFT Final Exam** **May 19, 2009**
4:00-6:00 p.m.

Instructions

- **Turn off all cell phones and watch alarms!** Put away iPods, etc.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.
- Do not use any other paper except this exam booklet and the one-page "cheat sheet" that you prepared. (Do *not* hand in your cheat sheet.)
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- If you use your calculator to do numerical calculations, be sure to show the setup leading to what you are calculating.
- Be prepared to show your UMass ID card when you **hand in your exam booklet to your own instructor or TA** ***as you exit the room.***

QUESTION	PER CENT	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	100	

The printed exam will have 1 question per 1–2 pages with space for work.

1. $(2 \times 10\% = 20\%)$ *The parts of this question are* not *related.*

   (a) Evaluate the indefinite integral:

   $$\int xe^{-2x}\,dx$$

   (b) Determine the derivative $f'(x)$ of the function

   $$f(x) = \int_1^{x^3+1} \sin\left(\sqrt{1+t^2}\right)\,dt.$$

2. $(2 \times 10\% = 20\%)$

   (a) Let R be the unbounded plane region in the first quadrant enclosed by the x-axis, the y-axis, and the graph of the function $y = \dfrac{1}{1+x^2}$. Compute the area of R by setting up and evaluating an appropriate improper integral. (Include in your work a rough sketch of the region R.)

   (b) Now let S be the bounded plane region enclosed by the x-axis, the y-axis, the line $x = 1$, and the graph of that same function $y = \dfrac{1}{1+x^2}$. A solid is obtained by rotating S around the x-axis. Compute the volume of this solid by setting up and evaluating an appropriate definite integral. (Include in your work a sketch that shows a typical cross-section, disk, or washer consistent with the integral you set up.)

3. $(2 \times 10\% = 20\%)$ *The parts of this question are* not *related.*

   (a) Determine whether the series $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^{2/3}}$ is absolutely convergent, conditionally convergent, or divergent.

   (b) Find the **interval** of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\frac{(x-2)^n}{n\,4^n}$.

4. Curve C has polar equation $r = \sin\theta + \cos\theta$.

   (a) (5%) Write parametric equations for the curve C.

   $$\begin{cases} x = \\ y = \end{cases}$$

   (b) (5%) Find the **slope** of the tangent line to C at its point where $\theta = \dfrac{\pi}{2}$.

   (c) (10%) Calculate the length of the arc for $0 \le \theta \le \pi$ of that same curve C with polar equation $r = \sin\theta + \cos\theta$.

5. (a) (12%) Determine the Taylor polynomial $T_2(x)$ of degree 2 for the function $f(x) = x^{1/7}$ centered at $a = 1$.

   (b) (8%) Suppose we were to use the approximation $f(x) \approx T_2(x)$. Obtain an upper bound on the error of this approximation when $0.7 \le x \le 1.3$. Give your answer rounded (up) to 4 decimal places.