Name (Last, First) —————————————— ID # ——————————

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Lecturer ———————————————————— Section # —————————

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UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

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Math 132   **DRAFT Exam 1**   **February 19, 2009**
7:00-8:30 p.m.

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Instructions

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- **Turn off all cell phones and watch alarms!** Put away iPods, etc.

- There are five (5) questions.

- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.

- Do not use any other paper except this exam booklet and the one-page "cheat sheet" that you prepared.

- Organize your work in an unambiguous order. Show all necessary steps.

- **Answers given without supporting work may receive 0 credit!**

- If you use your calculator to do numerical calculations, be sure to show the setup leading to what you are calculating.

- Be ready to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	100	

The printed exam will have 1 question per 1–2 pages with space for work.

1. $(2 \times 10\% = 20\%)$ Let

$$f(x) = \begin{cases} 6 & \text{if } x < 5, \\ 11 - x & \text{if } x \geq 5. \end{cases}$$

Let

$$G(x) = \int_2^x f(t) \, dt.$$

   (a) Calculate $G(3)$ and $G(8)$.
       [*Hint:* It may help to sketch first the graph of $f(x)$.]
   (b) Calculate $G'(3)$ and $G'(8)$.

2. $(4 \times 5\% = 20\%)$ Calculate each of the following. Explicitly show any relevant substitution or algebraic or trigonometric manipulation. In the case of a definite integral, give an exact value and *not* a numerical approximation.

   (a) $\displaystyle \int (3x + 8)^{132} \, dx$

   (b) $\displaystyle \int \frac{x}{\sqrt{1 - 9x^4}} \, dx$

   (c) $\displaystyle \int_1^2 \frac{e^{1/x}}{x^2} \, dx$

   (d) $\displaystyle \int \frac{\ln(\ln x)}{x} \, dx$. You may use the result that $\int \ln t \, dt = t \ln t - t + C$.

3. $(2 \times 10\% = 20\%)$ Soda leaks from a can at a rate of $r(t)$ milliliters per minute at time t minutes.

   (a) What does $\displaystyle \int_0^5 r(t) \, dt$ represent? (Be specific and include units in your answer.)

   (b) If, actually, $r(t) = \dfrac{1}{(2 + 3t)^2}$, exactly how much liquid will have leaked out after 10 minutes?

4. (20%) Calculate the exact area of the region enclosed by the curve $y = x^2$, the curve $y = (x - 1)^2$, and the x-axis.

   Begin by sketching the two graphs and the region. On your sketch include a typical rectangular strip of width Δx or Δy, whichever you are using.

5. (20%) The region R in the right half-plane (where $x \geq 0$) that is enclosed by the curve $y = \sin x$, the line $y = 1/2$, and the y-axis is rotated around the x-axis.

   Express as an integral the volume V of the solid obtained. (Do *not* evaluate this integral!) Include a two-dimensional and/or three-dimensional sketch to indicate how you arrive at that integral as your answer.