Name (Last, First) _____ **ID #** _____

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Lecturer _____ **Section (A, B, C, etc.)** _____

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 132 **DRAFT Final Exam** **May 19, 2008**
8:00-10:00 a.m.

Instructions

- **Turn off cell phones and watch alarms!** Put away cell phones, iPods, etc.
- There are six (6) questions.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.
- Do not use any other paper except this exam booklet and the one-page "cheat sheet" that you prepared. (Do *not* hand in your cheat sheet.)
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- If you use your calculator to do numerical calculations, be sure to show the setup leading to what you are calculating.
- Be prepared to show your UMass ID card when you **hand in your exam booklet to your own instructor or TA *as you exit the room.***

QUESTION	PER CENT	SCORE
1	16	
2	16	
3	18	
4	16	
5	16	
6	18	
TOTAL	100	

The printed exam will have 1 question per 1–2 pages with space for work.

1. $(2 \times 8 = 16\%)$ *The parts of this question are* not *directly related!*

   (a) If $f(1) = 12$, if the derivative f' is continuous, and if $\displaystyle\int_1^4 f'(x)\,dx = 17$, then what is the value $f(4)$?

   (b) Express the derivative $g'(x)$, for $0 < x < \pi/2$, as simply as possible if:

   $$g(x) = \int_{1/2}^{\sin x} \sqrt{1 - y^2}\,dy$$

2. $(2 \times 8\% = 16\%)$

   (a) Calculate the area of the bounded region R enclosed by the curves

   $$y = x^3 + 4, \qquad y = 4x^2 - 4x + 4.$$

   (b) The same region R as in (a)—enclosed by

   $$y = x^3 + 4, \qquad y = 4x^2 - 4x + 4$$

   —is rotated around the x-axis. Express the volume of the resulting solid as an integral but do ***not*** actually evaluate that integral. And do ***not*** attempt to "simplify" the function inside the integral.

3. $(3 \times 6 = 18\%)$ Use techniques of symbolic integration to evaluate:

   (a) $\displaystyle\int x\,e^{-x}\,dx$

   (b) $\displaystyle\int \frac{x}{\sqrt{x^2 + \frac{9}{16}}}\,dx$

   (c) $\displaystyle\int \frac{x^2}{\sqrt{1 + x^2}}\,dx$

4. A spiral has polar equation $r = e^{-2\theta}$ for $0 \le \theta < \infty$.

   (a) (6%) Write parametric equations for this spiral.

   (b) (10%) Find the length of the entire spiral for $0 \le \theta < \infty$.
   (*Hint:* This is easier to do if you work directly with the arc length formula for polar coordinates—and *not* the more general parametric formula.)

5. $(2 \times 8 = 16\%)$ Determine whether the series converges absolutely, converges conditionally only, or else diverges—and why.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3^n + \ln n}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3^n}{n^2 \cdot 2^n}$

6. (a) (6%) Starting with the Maclaurin series expansion of e^x, express the function e^{-x^2} as the sum of a power series. Use summation ($\sum$) notation.

   (b) (6%) Use (a) to express $\displaystyle\int_0^{0.4} \frac{e^{-x^2} - 1}{x}\, dx$ as the sum of a series of numbers. Use $\sum$ notation or give at least the first five terms of the series.

   (c) (6%) What is the least number of terms of that numerical series you would need so as to approximate that integral with error magnitude less than 10^{-8} ?

   [When answering this question, do ***not*** actually make the approximation, and do ***not*** evaluate the integral from (b)!]