Name (Last, First) ______________________________ **ID #** _____________

Signature ____

Lecturer ___________________________ **Section (A, B, C, etc.)** ________

UNIVERSITY OF MASSACHUSETTS AMHERST
DEPARTMENT OF MATHEMATICS AND STATISTICS

Math 132 **DRAFT Exam 3** **April 23, 2008**
7:00-8:30 p.m.

Instructions

- **Turn off cell phones and watch alarms!** Put away cell phones, iPods, etc.
- There are six (6) questions.
- Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.
- Do not use any other paper except this exam booklet and the one-page “cheat sheet” that you prepared.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**
- If you use your calculator to do numerical calculations, be sure to show the setup leading to what you are calculating.
- Be prepared to show your UMass ID card when you hand in your exam booklet.

QUESTION	PER CENT	SCORE
1	16	
2	16	
3	24	
4	12	
5	16	
6	16	
TOTAL	100	

The printed exam will have 1 question per 1–2 pages with space for work.

1. $(2 \times 8\% = 16\%)$
   (a) Express the repeating decimal $0.222\cdots = 0.\overline{2}$ as a series $\sum_{n=1}^{\infty} a_n$.
   Use summation ($\sum$) notation in your answer.
   (b) Determine the exact sum of this series (as a rational number).
   Use relevant methods for series—*not* some special formula you know about repeating decimals.

2. $(2 \times 8\% = 16\%)$
   (a) Find the **radius** of convergence of the power series $\sum_{n=1}^{\infty} \frac{3^n (x-2)^n}{n}$.
   (b) Now find the **interval** of convergence of the same power series.

3. $(3 \times 8\% = 24\%)$ Does the series converge? Why or why not? (Name the tests you use and indicate why the conditions needed for them to apply actually hold!)

   (a) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{3n^2 + 2n}{4n^2 - 5}$

   (b) $\sum_{n=1}^{\infty} \frac{\sin(2n)}{n^3}$

   (c) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

4. (a) (4%) Approximate $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}$ by using its **fourth** partial sum S_4.
   (b) (8%) Obtain an upper bound on the error of your approximation. Do this *without* finding the exact sum of any infinite series.

5. (16%) Find the Taylor polynomial $T_3(x)$—the sum of the first **four** terms of the Taylor series—for $f(x) = 8x^3 - 6x^2 + 5x + 3$ **centered at** $a = \mathbf{1}$.

6. $(12\% + 4\% = 16\%)$ Starting with the series expansion $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, which is valid for $|x| < 1 \ldots$

   (a) Represent $g(x) = \frac{x}{1 + 4x^2}$ by a power series in x.
   (b) For which values of x does your power series actually have sum $g(x)$?