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To apply this formula we must choose dv so that we can integrate it! Frequently, we choose u so that the derivative of u is simpler than u. If both properties hold, then you have made the correct choice.

Examples using strategy: $\int u\,dv = uv - \int v\,du$

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Integrating by parts gives

$$\int xe^x\,dx = xe^x - \int e^x\,dx = xe^x - e^x + C.$$

Example Evaluate $\int \ln x \; dx$.

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Solution Let

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$$\mathbf{u} = \ln \mathbf{x} \qquad \mathbf{dv} = \mathbf{dx}.$$

Then

$$\mathbf{du} = \frac{\mathbf{1}}{\mathbf{x}}\mathbf{dx} \qquad \mathbf{v} = \mathbf{x}.$$

Example Evaluate $\int \ln x\ dx$.

Solution Let

$$\mathbf{u = \ln x \qquad dv = dx.}$$

Then

$$\mathbf{du = \frac{1}{x}dx \qquad v = x.}$$

Integrating by parts, we get

$$\int \ln x\,dx = x\ln x - \int x\frac{dx}{x}$$

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$$= t^2 e^t - 2te^t + 2e^t + C_1 \ \text{where} C_1 = -2C.$$

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We put Equation **(3)** into Equation **(2)** and we get $\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x - \int e^x \sin x\,dx$. This can be regarded as an equation to be solved for the unknown integral.

Example Evaluate $\int e^x \sin x \, dx$.

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$$\int e^x \sin x\,dx = \frac{1}{2}e^x(\sin x - \cos x) + C.$$

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Integrating such functions involve several techniques and strategies which we will describe today.

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$$\cos^2(\theta) + \sin^2(\theta) = 1.$$

$$\sec^2(\theta) - \tan^2(\theta) = 1.$$

$$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$$

$$\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$$

Example Evaluate $\int \cos^3 x \, dx$.

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Let $\mathbf{u} = \sin \mathbf{x}$, so $\mathbf{du} = \cos \mathbf{x} \ \mathbf{dx}$.

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Let $\mathbf{u} = \sin \mathbf{x}$, so $\mathbf{du} = \cos \mathbf{x}\; \mathbf{dx}$. So, we get

$$\int (1 - u^2)\, du = u - \frac{1}{3} u^3 + C$$

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$$= \sin x - \frac{1}{3}\sin^3 x + C.$$

Example Find $\int_0^{\pi} \sin^2 x\, dx$.

Solution Here we use the **double angle formula**:

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$$= \frac{1}{2}(\pi - \frac{1}{2}\sin 2\pi) - \frac{1}{2}(0 - \frac{1}{2}\sin 0) = \frac{1}{2}\pi.$$

Here we mentally made the **substitution $\mathbf{u = 2x}$** when integrating $\cos 2x$.

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Then **substitute $\mathbf{u = \tan x}$**.

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Also, don't forget that $\frac{d}{dx} \tan x = \sec^2 x$ and $\frac{d}{dx} \sec x = \sec x \tan x$.

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In the first integral we mentally **substituted** $\mathbf{u = \tan x}$ so that $\mathbf{du = \sec^2 x \, dx}$

To evaluate the integrals **(a)** $\int \sin mx \cos nx \ dx$, **(b)** $\int \sin mx \sin n; dx$, or **(c)** $\int \cos mx \cos nx \ dx$, use the corresponding identity:

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Table of Trigonometric Substitution

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$\mathbf{x} = \mathbf{a}\sin\theta, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$\mathbf{x} = \mathbf{a}\tan\theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$\mathbf{x} = \mathbf{a}\sec\theta, \quad \mathbf{0} \leq \theta < \frac{\pi}{2}$ or $\pi \leq \theta < \frac{3\pi}{2}$	$\sec^2\theta - 1 = \tan^2\theta$

$\sqrt{a^2 - x^2}$, $x = a \sin\theta$, $1 - \sin^2\theta = \cos^2\theta$

Example Find the area enclosed by the ellipse $\frac{x^2}{a} + \frac{y^2}{b} = 1$.

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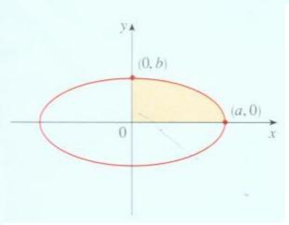

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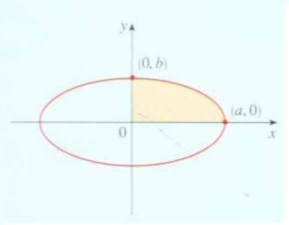

Solution Solving for y gives
$y = \frac{b}{a}\sqrt{a^2 - x^2}$ and $A = \frac{4b}{a}\int_0^a \sqrt{a^2 - x^2}\, dx$

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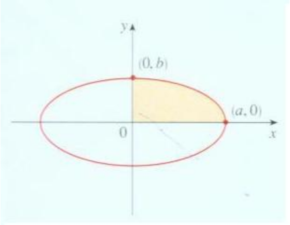

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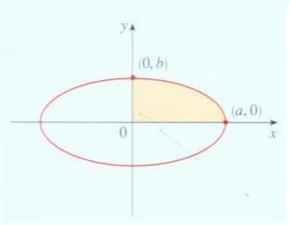

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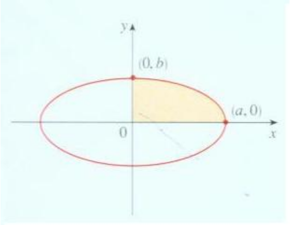

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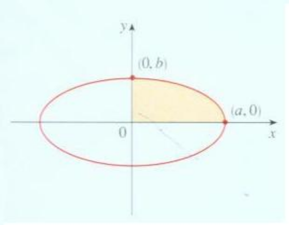

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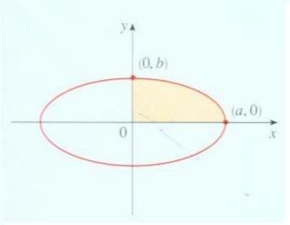

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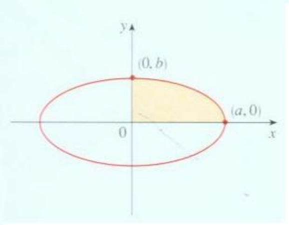

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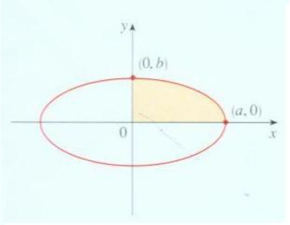

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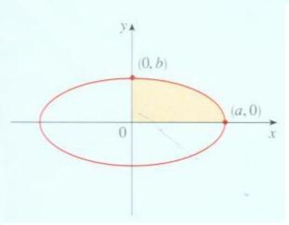

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Substitute $\mathbf{x} = \mathbf{a}\sin\theta$, $\mathbf{dx} = \mathbf{a}\cos\theta\, \mathbf{d}\theta$ and use

$\sqrt{a^2 - x^2} = a\cos\theta$.

$\int \sqrt{a^2 - x^2}\, dx = \int a\cos\theta \cdot a\cos\theta\, d\theta$

$= a^2 \int \cos^2\theta\, d\theta = a^2 \int \frac{1}{2}(1 + \cos 2\theta)\, d\theta = \frac{1}{2}a^2(\theta + \frac{1}{2}\sin 2\theta)$.

$A = \frac{4b}{a}\int_0^a \sqrt{a^2 - x^2} = 2ab\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi} = \pi ab$.