

## Computational Session 10/21/09

The next thing we are going to learn about is Gibbs sampling and other ways to sample from complicated distributions. Today's class consists of two things:

1. Read an article: "Explaining the Gibbs sampler." Please read at least a little of the article in class.

(The article is at [www.math.umass.edu/~jstauden/ExpGibbsSamp.pdf](http://www.math.umass.edu/~jstauden/ExpGibbsSamp.pdf))

2. Use Gibbs sampling to sample from a not-so-complicated distribution as an illustration.

1. As a start, we'll use Gibbs sampling to sample from:  $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \sim N \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right\}$ .

(Note that the R function `rmvnorm()` in the `mvtnorm` library can do this directly.)

The Gibbs sampler samples from this distribution by iterating the following:

$$\begin{aligned} x_1^{(m)} &\sim p(X_1|X_2 = x_2^{(m-1)}) \\ x_2^{(m)} &\sim p(X_2|X_1 = x_1^{(m)}) \end{aligned}$$

for  $m = 1, \dots, M$ . (Note that you need a starting value for  $x_2$ . Below we'll use  $x_2 = -100$ .) To implement the result, you can use that  $x_1|x_2$  is  $N(\rho x_2, 1 - \rho^2)$ . (You should be aware of a more general result that allows you to get a conditional distribution from a joint distribution when the joint distribution is multivariate normal.) Please implement the algorithm. There is code at [www.math.umass.edu/~jstauden/gibbs1.R](http://www.math.umass.edu/~jstauden/gibbs1.R) to help you do the following.

- (a) Implement this algorithm in R and plot the results.
- (b) How well or poorly does the sample from this algorithm estimate  $E(X_1), E(X_2)$  and  $Cov \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ ? Is there a key tuning parameter to improve or degrade those estimates? Is -100 a good starting value? What would a better one be?
- (c) Use the results from this algorithm to estimate  $Pr(x_1 < 0.2, x_2 < 2)$  and  $Pr(X < 1)$ . Please also estimate the Monte Carlo standard error of your estimates.
- (d) Write your own code to estimate a 95% interval for  $E(X_1)$ . (Hint: use the `quantile()` function in R.)