Math 411–Spring 2006
PRACTICE FINAL EXAM

1. Give an example of a group G such that
   (a) G has no nontrivial proper subgroups.
   (b) G has order 6 and **not** isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_3$.
2. Let $\phi : S_3 \to Q_8$ be a non-trivial homomorphism.
   (a) Find the kernel $\mathrm{Ker}(\phi)$.
   (b) How many different non-trivial homomorphisms $\phi : S_3 \to Q_8$ are there?
3. Which of the following groups are isomorphic and which are not? Give a short reason for each answer.
   (a) S_3
   (b) $\mathrm{GL}(2, \mathbb{Z}_2)$
   (c) $\{z \in \mathbb{C} \mid z^6 = 1\}$
   (d) $\mathbb{Z}_{60}/H$, where H is the subgroup generated by $[10] \in \mathbb{Z}_{60}$
4. How many non-isomorphic Abelian groups of order 120 are there? Write each such group as a product of cyclic groups.
5. Let S_4 act on itself by conjugation.
   (a) Find the size of the orbit $\mathcal{O}_{(12)}$.
   (b) Find the elements of the centralizer subgroup $C_{(12)}$.
6. Consider the action of the group $G = \mathbb{R}^*$ on the set $X = \mathbb{R}^2$ given by $r \cdot (a, b) = (ar, b/r)$ for $r \in \mathbb{R}^*$ and $(a, b) \in \mathbb{R}^2$.
   (a) Show that this defines a group action.
   (b) Describe the orbits of the action. (Hint: most of the orbits are hyperbolas.)
7. How many essentially different tetrahedra can one build if 6 black, 6 white, and 6 red rods (all of the same size) are available?