Math 411 — Spring 2006
Review for exam 2

Format and Coverage: You will be asked to give some definitions, proofs, examples, and counter-examples. The exam covers sections 1.4, 2.1 – 2.5, 3.1, 3.2 excluding some material that we did not discuss in class.

The following is a practice exam, which is also your homework for the week (do not submit).

1. Give the definition of a coset. If G is a group and H is its subgroup prove that two cosets aH and bH either do not intersect or coincide (try to prove it without looking at the textbook/notes).

2. Let $f\colon G \to G'$ be a group homomorphism. Show that the order of an element $a \in G$ is divisible by the order of its image $f(a) \in G'$.

3. Does there exist a homomorphism $\phi\colon \mathbb{Z}_6 \to \mathbb{Z}_3 \times \mathbb{Z}_2$ that maps $[2]$ to $([1],[1])$? Justify.

4. Let $\phi\colon \mathbb{Z}_9 \to S_3$ be the unique homomorphism that maps $[1]$ to the 3-cycle (123). Determine the kernel and the image of ϕ.

5. Describe all possible **onto** homomorphisms from D_4 to $\mathbb{Z}_4$.

6. Let G be a group.

   (a) Show that $\phi \circ T_g = T_{\phi(g)} \circ \phi$ for any $\phi \in Aut(G)$ and any $g \in G$.

   (b) Prove that $Inn(G)$ is a normal subgroup of $Aut(G)$.

7. Show that $V_4 \cong \mathbb{Z}_2 \times \mathbb{Z}_2$.

8. (Bonus) Prove that $S_4/V_4 \cong S_3$. Hint: There are three possible partitions of the set $\{1,2,3,4\}$ into 2-element subsets: $a = \{1,2\} \cup \{3,4\}$, $b = \{1,3\} \cup \{2,4\}$, and $c = \{1,4\} \cup \{2,3\}$. Each permutation in S_4 induces a permutation on the set $\{a,b,c\}$, i.e. defines an element of S_3. Use the first isomorphism theorem.