DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS
MATH 411 May 23, 2006
FINAL EXAM

Your Name: ____________________________________

Your Section: 9:30 AM 11:15 AM (circle one)

This exam paper consists of 8 questions. It has 8 pages. You have 2 hours.
Each answer must be **justified**. No calculators, books or notes are allowed!

1.	(10)	____________	
2.	(15)	____________	
3.	(15)	____________	
4.	(15)	____________	
5.	(15)	____________	
6.	(15)	____________	
7.	(15)	____________	
8.	(20)	____________	(*bonus*)
TOTAL	(120)		

1. *(10 pts)* Give an example of a group G such that

   (a) G is non-Abelian with non-trivial center.

   (b) G is Abelian of order 12, but not cyclic.

2. Does there exist a homomorphism $\phi : \mathbb{Z}_{18} \to \mathbb{Z}_4 \times \mathbb{Z}_6$ such that $\phi(3) = (3, 3)$? If yes, construct an example; if no, give a reason.

3. How many non-isomorphic Abelian groups of order 180 are there? Write each such group as a product of cyclic groups.

4. Which of the following groups are isomorphic? Give a short reason for each answer.

   (a) Q_8 (the quaternions)

   (b) $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$

   (c) $\{z \in \mathbb{C} \mid z^8 = 1\}$

   (d) $\mathbb{Z}_2 \times V_4$

   (f) $\mathbb{Z}_8$

   (g) D_4

5. Let S_5 act on itself by conjugation.

   (a) Find the number of orbits of the action (i.e. conjugacy classes).

   (b) Find the size of the orbit $\mathcal{O}_{(12)}$.

   (c) Find the size of the centralizer subgroup $C_{(12)}$.

6. Consider the action of the group $G = \mathbb{R}^*$ on the set $X = \mathbb{R}^2$ given by $r \cdot v = rv$ (scaling) for $r \in \mathbb{R}^*$ and $v \in \mathbb{R}^2$.

   (a) Show that this defines a group action.

   (b) Describe the orbits of the action. (Make sure your description agrees with the fact that X is partitioned into orbits.)

7. How many essentially different colorings of the faces of the regular tetrahedron are there if

   (a) 4 colors are available and no two faces are allowed to have the same color?

   (b) 4 colors are available and faces are allowed to have the same color?

8. (*Bonus.*) Let G be a group of order pq, where p and q are distinct primes. Prove that the center $Z(G)$ cannot be a proper non-trivial subgroup of G.