**Practice for Exam 1**

1. Find gcd(9750,59400).

2. Find all complex solutions to $z^4 = -1$.

3. (15 points) Solve a system of congruences:

$$\begin{cases} 3x & \equiv 4 \mod 5 \\ 2x & \equiv 3 \mod 9 \end{cases}$$

4. (a) Give the definition of a subgroup.

   (b) Let $\Lambda(2, \mathbb{R})$ be the set of all doubly stochastic matrices in $GL(2, \mathbb{R})$ (that is, all row sums are 1 and all column sums are 1). Show that $\Lambda(2, \mathbb{R})$ is a subgroup of the multiplicative group $GL(2, \mathbb{R})$.

5. (20 points) Show that the unit circle

$$\{z = x + iy \in \mathbb{C} \mid x^2 + y^2 = 1\}$$

   is a subgroup of the multiplicative group $\mathbb{C} \setminus \{0\}$.

6. (a) Give the definition of the order of a group $G$.

   (b) Give the definition of the order of an element $a$ in a group $G$.

   (c) Find the order of

$$\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$$

   in the multiplicative group $GL(2, \mathbb{R})$.

7. Is $U(12)$ a cyclic group?

8. Let $G = \langle a \rangle$ be a cyclic group of order 18. Let $H$ be a subgroup of $G$ generated by the element $a^2$, and $K$ a subgroup of $G$ generated by the element $a^3$. Find the orders of the groups $H$, $K$, $H \cap K$.

9. (a) Give the definition of the center $Z(G)$ of a group $G$.

   (b) Find the center of $D_4$.