Math 233

1. (a) ( 6 points) Find the line of intersection of the planes given by equations $2 x-3 y+4 z+5=$ 0 and $y-z=7$.
(b) Where does the line from part (a) intersect the plane $x+y+z=0$ ?
2. (a) Given the points $A=(1,0,0), B=(0,1,0)$ and $C=(0,0,1)$, find the point $P$ on the line segment $\overrightarrow{A B}$ that is closest to $C$.
(b) Find the area of the triangle with vertices $A, B$, and $C$.
(c) Find the plane that contains the points $A, B$, and $C$.
3. For the surface $z^{2}=x^{2}+y^{2}-1$, do the following
(a) Write down the equation of the slice (intersection) of this surface with the plane $z=3$, and use it to completely describe the curve.
(b) Sketch the slices of this surface in all three coordinate planes.
(c) Find a vector valued function $\vec{r}(t)$ that gives the cuurve in part (a).
4. Consider the line $L_{1}$ given by $x=4+t, y=3+t, z=1+2 t$ and the line $L_{2}$ given by $x=1-t, y=2 t, z=1+t$, and also the point $P=(3,2,-1)$.
(a) Find a parametric equation of a line $L$ that passes through $P$ and is perpendicular to both $L_{1}$ and $L_{2}$.
(b) Show that $P$ lies on $L_{1}$ and find the point $Q$ at which $L$ and $L_{2}$ meet.
(c) What is the distance between lines $L_{1}$ and $L_{2}$ ? Why?
5. The acceleration vector of a particle moving in space at a time $t$ is $\mathbf{a}(t)=-2 t \mathbf{i}+4 \mathbf{j}$.
(a) Find the position $\mathbf{r}(t)$ of the particle as a function of $t$, if at the time $t=0$ the velocity vetor is $\mathbf{v}(0)=\langle 3,0,4\rangle$ and at time $t=3$ the particle is at the point $(0,1,0)$.
(b) Find an equation of the tangent line to the curve at the point $(0,1,0)$.
(c) Find the length of the trajectory traveled from time $t=0$ to time $t=2$.
