DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS

MATH 233 **FINAL EXAM** **Spring 2006**

NAME:______________________________ Student ID# ____________________

Section Number:____________________ Instructor's Name: ____________________

In problems that require reasoning or algebraic calculation, it is not sufficient just to write the answers. You must explain how you arrived at your answers, and show your algebraic calculations. Definite integrals must be solved symbolically, not by calculator.

You can leave answers in terms of fractions and square roots, but if approximate numerical answers are used, they should be rounded off to 4 significant figures.

1.	(18)	________
2.	(17)	________
3.	(12)	________
4.	(18)	________
5.	(15)	________
6.	(10)	________
7.	(10)	________
Total		________

Perfect Paper $\longrightarrow$ 100 Points.

There are nine pages, including this one, in this exam and seven problems. The last page is a sheet of formulas. Make sure you have all the pages before you begin!

1. For a)-c), a particle's position is given by the vector function $\mathbf{r}(t) = \langle t^2, 2t + 1, 5t \rangle$.

a) (6 points) Find the point where the particle's velocity and acceleration are orthogonal.

b) (6 points) What is the speed of the particle at the time $t = 2$?

c) (6 points) Find the point (x, y, z) where the particle hits the surface $z = \sqrt{x + y}$.

2. a) (7 points) Let $f(x, y, z) = x^2y^3z^4$. Find the maximal rate of change of f at the point $(1, 1, -1)$ and the direction in which it occurs.

b) (6 points) Find the tangent plane of the level surface $x^2y^3z^4 = 1$ at $(1, 1, -1)$.

c) (4 points) Is the tangent plane from b) parallel to the xy plane? Explain your answer.

3. (12 points) Find the volume of the solid under the surface $z = x^2 + y^2$ and above the region in the xy plane between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

4. (18 points) Find the absolute maximum and minimum values of

$$f(x, y) = 3x^2 - 2xy + y$$

on the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 4)$ (including the interior). Also indicate the points at which f takes those maximum and minimum values.

5. a) (8 points) (Show that the vector field

$$\mathbf{F}(x, y, z) = 3x^2y^2z\,\mathbf{i} + (z\cos(yz) + 2x^3yz)\mathbf{j} + (y\cos(yz) + x^3y^2)\mathbf{k}$$

is conservative by finding a potential function $f(x, y, z)$.

b) (7 points) Use your answer from part a) to compute

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

where the curve C is given by the vector function $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$, $0 \le t \le \pi$. (Even if you did not get a), you can still get partial credit by explaining how to use a potential function to evaluate this integral.)

6. (10 points) Compute the line integral

$$\int_C x^2 y\, dx + x\, dy,$$

where C is the part of the parabola $y = x^2$ from $(0, 0)$ to $(1, 1)$.

7. (10 points) Use Green's theorem to compute $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = 2xy\,\mathbf{i} + e^{y^2}\,\mathbf{j}$ and the path C consists of three line segments: from $(0,0)$ to $(2,-2)$, from $(2,-2)$ to $(2,2)$, and from $(2,2)$ back to $(0,0)$.

FORMULAS

$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}||\mathbf{b}| \cos\theta$

$|\mathbf{a}| = \sqrt{\mathbf{a} \cdot \mathbf{a}} = \sqrt{a_1^2 + a_2^2 + a_3^2}.$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta =$ area of parallelogram with sides $\mathbf{a}$, $\mathbf{b}$.

Plane through (x_0, y_0, z_0) with normal vector $\langle a, b, c \rangle$:

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Line through (x_0, y_0, z_0) with direction vector $\langle a, b, c \rangle$:

$$\begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \\ z &= z_0 + ct \end{aligned}$$

$D_{\mathbf{u}} f(a, b, c) = \nabla f(a, b, c) \cdot \mathbf{u}.$

Green's theorem:

$$\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

if C is the positively oriented boundary of the region D.