

# Math 421 – Practice Final

## May 6, 2009

1. Find the Taylor series or Laurent series of  $f(z) = \frac{2}{z-1}$  in the following domains.

- (a)  $D_1 = \{z \mid |z| < 1\}$ .
- (b)  $D_2 = \{z \mid 1 < |z| < \infty\}$ .
- (c)  $D_5 = \{z \mid 1 < |z-2| < \infty\}$ .

2. We know  $f(z) = \sin(\sin z) - \sin z$  has a zero at  $z_0 = 0$ . Find its order.

3. Find the residues of  $f$  at all its isolated singular points.

- (a)  $f(z) = \frac{z^2}{(z^2+1)^2}$ .
- (b)  $f(z) = \frac{z^n}{(z+3)^n}$ , where  $n$  is a positive integer.
- (c)  $f(z) = \frac{\cot z}{z}$ .

4. Evaluate the following contour integrals.

- (a)  $\oint_{|z|=6} \frac{\cos z}{z^2(z-\pi)^3} dz$ .
- (b)  $\oint_{|z|=10} \frac{z}{e^z-1} dz$ .
- (c)  $\oint_{|z|=1} e^{(\frac{1}{z})} \sin(\frac{1}{z}) dz$ .

5. Evaluate the following integrals.

- (a)  $\int_0^{2\pi} \frac{\sin^2 \theta}{5+4\cos \theta} d\theta$ .
- (b)  $\int_0^\infty \frac{x^2}{(x^2+4)(x^2+9)} dx$ .
- (c)  $\int_0^\infty \frac{x \sin x}{(x^2+1)(x^2+4)} dx$ .

6. Determine the values of  $\Delta_C \arg f(z)$  for the function  $f$  and positively oriented contour  $C$ .

- (a)  $f(z) = \frac{4z^3+3}{z}$ ;  $C = \{z \mid |z| = 1\}$ .
- (b)  $f(z) = \tan z$ ;  $C = \{z \mid |z| = 10\}$ .

7. Determine the number of roots, counting multiplicities, of the following equations in the given regions.

- (a)  $z^5 + z^2 + 10z + 3 = 0$ ;  $D_1 = \{z \mid |z| < 1\}$ ,  $D_2 = \{z \mid |z| < 2\}$ , and  $D_3 = \{z \mid 1 \leq |z| < 2\}$ .
- (b)  $5z^6 = \cos z + 1$ ;  $D = \{z \mid |z| < 1\}$ .