MATH 611 ABSTRACT ALGEBRA I
SAMPLE FINAL EXAM

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Throughout, R is a commutative ring and unless otherwise stated, M, N are R-modules.

(1) a) Describe the module structure of $\mathrm{Hom}_R(M, N)$, the set of all R-linear maps from M to N.
   b) Prove that $\mathrm{Hom}_R(R, M)$ is isomorphic to M as R-module.
(2) Define what it means for M to be a Noetherian module and give 2 other equivalent conditions for it.
(3) a) Let $f : R \to S$ be a ring homomorphism and let N be an S-module. Describe how we think of N as an R-module.
   b) Suppose S is finitely generated as R-module and N if finitely generated as S-module. Show that N is finitely generated as R-module.
   c) If M is an R-module how do you extend scalars to get an associated S-module $N = M_S$?
(4) What does it mean for S to be an R-algebra? What is a synonym for $\mathbb{Z}$-algebra?
(5) Show that M is a finitely generated R-module if and only if M is isomorphic to a quotient of R^n for some integer $n > 0$¿
(6) Show that if N, N' are submodules of M, then $(N+N')/N \approx N'/(N \cap N')$.
(7) Suppose I is an ideal of R, and let $IM = \{\sum_{i=1}^{k} r_i m_i \mid r_i \in I, m_i \in M, k \geq 0\}$. Show that $M/IM \approx (R/I) \otimes_R M$.
(8) For a f.g. module M, over a PID, describe the structure of $M/\mathrm{Tor}(M)$.