UMASS AMHERST MATH 461 F. HAJIR

SAMPLE MIDTERM EXAM = HOMEWORK 7

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1. DEFINITIONS

a. The protractor postulate of Neutral Geometry states that

b. We say that triangle ABC is congruent to triangle DEF under the mapping $A \mapsto D$, $B \mapsto E$, $C \mapsto F$ if

c. If S is a set of points in the plane, we say that S is convex if

d. If $\vec{a}, \vec{b}, \vec{c}$ are three distinct rays sharing a vertex, we say that $\vec{b}$ lies between $\vec{a}$ and $\vec{c}$ if

e. Give a complete description of the Cartesian Plane model, as a model of Neutral Geometry. I'm not asking you to prove that it is a model of Neutral Geometry, only that you give a precise description of the all the primitives and definitions needed for statements of the postulates of Neutral Geometry.

f. The Triangle Inequality states that

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2. TRUE/FALSE (You do not need to justify your answer).

a. Every model of Euclidean Geometry is isomorphic to the Cartesian plane.

b. Taxi-Cab Geometry satisfies the AAS Theorem.

c. The Fano Plane satisfies the Hyperbolic Parallel Postulate.

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3. SHORT ANSWER

a. Suppose S and T are subsets of the plane and P is a point in the intersection $S \cap T$. Suppose further that P is a passing point of S and also a passing point of T. In such a situation, is it always the case that P is a passing point of $S \cap T$? Either give a proof, or give a counterexample.

b. Suppose we have a triangle one of whose angles measures 45 degrees. We know two more facts about this triangle. It has one side of length 1 and another side of length 2. Is this enough information to determine the triangle up to a congruence? In other words, are all triangles satisfying these three properties congruent to each other? Discuss why or why not.

c. Suppose M is a model of Incidence Geometry. We define the dual of M, labeled $\widehat{M}$, as follows. The set of points of $\widehat{M}$ is the set of lines of M, and the set of lines of $\widehat{M}$ is the set of points of M. Moreover, a point and line in $\widehat{M}$ are incident if and only if the corresponding line and point are incident in M.

(i) Describe the dual of the three-point plane. Is it a model of IG?

(ii) What is the dual of the four-point plane? Is it a model of IG?

d. Sketch the "unit circle" in taxicab geometry. In other words, sketch the locus of all points (x, y) that satisfy $d((x, y), (0, 0)) = 1$ where $d(A, B)$ is the taxicab distance between A and B.

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4. PROOFS

a. State and prove the Exterior Angle Theorem.

b. Consider a model defined as follows. Points, lines and distance are defined exactly as in the case of the Cartesian plane, but angle measures are given as follows. Given an angle defined by two rays $\vec{a}$ and $\vec{b}$ with common vertex O, construct the unique point A on the ray $\vec{a}$ such that $OA = 1$, and similarly construct the unique point B on $\vec{b}$ such that $OB = 1$. Then define the measure of the angle $\angle\vec{a}\vec{b}$ to be $m(\angle\vec{a}\vec{b}) = 90AB$.

Discuss whether this is a model for Neutral Geometry.

c. Suppose we define an alternate axiomatic system, in which the SAS postulate is replaced by the AAS postulate. Show that in this axiomatic system, SAS is a theorem. In other words, show that AAS congruence (along with all the other 8 postulates of Neutral Geometry) implies SAS congruence.
