UMASS AMHERST MATH 461 F. HAJIR

SAMPLE FINAL EXAM / EXTRA CREDIT HW

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1. $\bowtie$ DEFINITIONS

a. State the names of the postulates of Neutral Geometry.

b. Define polygon, convex-polygon, and what it means for two polygons to be similar.

c. State the law of cosines.

d. Define the circumcenter of a triangle.

e. Define what we mean by an area function and state the Euclidean Area Postulate.

f. Define what it means for a line to be tangent (secant) to a circle.

g. State the Two Transversals Theorem, the Parallel Projection Theorem, and the Angle Bisector Proportions Theorem.

2. $\bowtie$ TRUE/FALSE

a. For a fixed $n \geq 3$, any two regular n-gons are congruent to each other.

b. If $\triangle ABC$ and $\triangle DEF$ satisfy $AB/AC = DE/DF$, then they are similar.

c. If $\mathcal{C}$ is a circle with center at O and P is exterior to $\mathcal{C}$, then among all points on $\mathcal{C}$, exactly one of them is closest to P.

d. If $\mathcal{C}$ is a circle with center at O and P is exterior to $\mathcal{C}$, then there are exactly two points Q, Q' on $\mathcal{C}$ such that the lines $\overleftrightarrow{PQ}$ and $\overleftrightarrow{PQ'}$ are tangent to $\mathcal{C}$.

3. $\bowtie$ SHORT ANSWER

a. Compute the area of a triangle which has side lengths $7, 8, 9$. (That's why 6 was afraid of 7, of course).

b. For the triangle above, if θ is the measure of the smallest angle, compute $\cos\theta$.

c. Let $\mathcal{C}$ be the unit circle, i.e. the center is at $(0,0)$ and the radius is 1. Let P be the point $(5/3,0)$. Find a point Q on the unit circle such that the line passing through P and Q is tangent to $\mathcal{C}$.

4. $\bowtie$ Proofs (all in Euclidean Geometry)

a. Consider the line segment $\overline{AM}$ in $\triangle ABC$, where M is the midpoint of $\overline{BC}$. Prove that $AM < (AB + AC)/2$. For a hint, look at this footnote. For lack of a hint, don't peek. [1]

b. Suppose x, y, w, z are four positive real numbers such that $x^2 + y^2 = w^2 + z^2$. Is it true that a quadrilateral $ABCD$ with side lengths $AB = x, BC = y, CD = w, DA = z$ must be cyclic? If true, prove it, if false, provide a counterexample.

c. For a triangle $\triangle ABC$, draw lines passing through each vertex parallel to the opposite side. Any pair of these lines meets a point; for the three pairs of lines, call the intersection points D, E and F. Show that A, B, C are the midpoints of the triangle $\triangle DEF$.

[1] *Hint: when you draw the line from A to M, why not keep going for a while?*