UMASS AMHERST MATH 300 FALL '08, F. HAJIR

EXAM 1 REVIEW

The material you should know for Exam 1 is as follows: everything that appears in HW1, HW2, HW3 and HW4, including the reading specified in those homework assignments. The exam will have three parts: 1. Defintions, 2. Short Answer, and 3. Problems. The problems will usually but not always require you to write a cogent, concise and correct proof. Some of the problems will be statements that were already proved in class or are taken directly from homework. But at least some of the problems will require you to prove a statement that has not been presented to you before, or to find a counterexample to a statement.

For the definitions, it is important to be extremely precise. For instance if I ask you define what it means for $f : X \to Y$ to be surjective, the response "f is surjective means that for all $y \in Y$, there exists $x \in X$ such that $f(x) = y$ receives full credit, and "everyting in Y gets hit by somebody in X" receives only partial credit because "gets hit by" is not sufficiently precise.

Here begineth the sample exam.

The points will be distributed approximately as follows: 25% Definitions, 25% Short Answer, and 50% Problems.

You may wish to take this exam in a quiet room without notes under a time constraint (or not, this is just a suggestion; it may be a good suggestion for some students and not so good for others). The actual exam will be somewhat similar, but not identical!!, to this one. This practice exam is LONGER than the actual exam.

Sample Exam 1

1. Definitions

A set is

A set X is finite means that

A bijection from X to Y is

If $f : X \to Y$ and $g : Y \to Z$ are maps, then the composite map $g \circ f$ has source .... and target .... and is defined by

A set X is equal to a set Y means that

A set X is a subset of a set Y means that

The intersection of X and Y is defined by

The power set $\mathcal{P}(X)$ is

A partition of a set X is

We say that two statements P and Q are equivalent if

The direct or Cartesian product of X and Y is the set $X \times Y =$

A map $f : X \to Y$ is invertible if

2. Short Answer

The converse of $-P \Rightarrow -Q$ is

The negation of $(P \wedge Q)$ is $-(P \wedge Q) =$..........

Let R be the statement: Whenever it rains, my car gets wet. State the negation of R.

Determine whether $(-P \vee -Q) \Leftrightarrow -(P \wedge Q)$ is a tautology.

Write down two sets, X and Y, say, which are equivalent, but not equal.

Construct the truth table for $(P \Rightarrow Q) \Rightarrow (Q \Rightarrow P)$ and determine whether it is equivalent to $(Q \Rightarrow P)$.

Give a bijection $[0,1] \to [0,1]$ other than the identity map.

For each statement below, Indicate whether it is True or False.

If X is equal to Y, then $X \subseteq Y$ and $Y \subseteq X$.

If $f : X \to Y$ is surjective and $g : X \to Y$ is injective then $g \circ f$ is bijective.

If X and Y are equivalent sets, then X is finite if and only if Y is finite.

If X is an infinite set, then every subset of X is equivalent to X.

If X is a set, then $\{\} \subseteq X$.

If $X \subseteq Y$ and $Y \subsetneq Z$, then $X \subsetneq Z$.

If X is finite, then $\mathcal{P}(X)$ is also finite.

The set $\mathbb{N} = \{1, 2, 3, \ldots\}$ is equivalent to the set $\{y \in \mathbb{Z} | y \geq 1984\}$.

If X is a non-empty subset of a set Y, then $\{X, Y \setminus X\}$ is a partition of Y.

If $f : X \to Y$ is surjective and $g : Y \to Z$ is surjective, then $g \circ f$ is surjective.

3. Problems

1. Prove that if X is a set, then there is an injection $f : X \hookrightarrow \mathcal{P}(X)$.

2. Suppose A, B, C are sets. (a) Show that $C \setminus (A \cup B) \subseteq C \setminus A$.

(b) Give explicity three sets A, B, C such that $C \setminus (A \cup B)$ is a proper subset of $C \setminus A$.

3. Suppose $f : X \to Y$ and $g : Y \to Z$ are given maps and let $h = g \circ f$.

(a) Prove that if h is surjective, then so is g.

(b) Specify an example of X, Y, Z, f, g as above so that g is surjective but f isn't. Drawing a clear and detailed picture will suffice.

4. Let $f : X \to Y$ be a map. Suppose there exist functions $g : Y \to X$ and $h : Y \to X$ such that $g(f(x)) = x$ for all $x \in X$ and $f(h(y)) = y$ for all $y \in Y$.

i) Show that $g = h$.

ii) Show that g is bijective.