UMASS AMHERST MATH 300 FALL '05, F. HAJIR

HOMEWORK 2: LOGIC AND SETS

1. READING

You should read all of Chatper 1 in Gilbert/Vanstone as well as Part II of Farshid's class notes.

2. PROBLEMS FROM GILBERT/VANSTONE CHAPTER 1

Exercise Set 1 (p. 20): 1-13,25,41-44, 56,58,60, 64, 66,69.
Problem Set 1 (p. 22): 74,76.

3. PROBLEMS FROM FARSHID'S BRAIN

1. Prove that $P \Rightarrow (P \vee Q)$ is a tautology, i.e. its truth table has value "True" in all cases.

2. (a) Prove that if X is an infinite set, then X has an infinite number of subsets.
(b) Prove that if a set X has a finite number of subsets, then X is a finite set.
(c) What is the relationship between the Proposition in (b) and the Proposition in (c)? Can you prove (c) using (a) and (b)?

3. (a) Suppose the union of ten sets $A_1 \cup A_2 \cup \cdots \cup A_{10}$ equals A_1. What can you conclude about these sets?
(b) Suppose the intersection of ten sets $B_1 \cap B_2 \cap \cdots \cap B_{10}$ is B_1. What can you conclude about these sets?

4. Consider the sets $A = \{0, 1\}$, $B = \{a, b, c\}$. List the elements of the sets $A \times A$, $A \times B$, $B \times A$, $A \times B \times A$.

5. Prove that if $P \Rightarrow Q$ and $Q \Rightarrow R$ and $R \Rightarrow P$, then P, Q, R are all pairwise equivalent.

4. EXTRA CREDIT

1. Construct a sequence of sets $S_1, S_2, S_3, \ldots$ (one for each natrual number) such that for any finite subset $\{i_1, \ldots, i_n\} \subset \mathbb{N}$ of the natrual numbers, the intersection $S_{i_1} \cap \cdots \cap S_{i_n}$ is an infinite set, but $\cap_{n \geq 1} S_n = \{\}$.