

Midterm Practice Problems Math 235 Fall 2007

1. Write the following system of equations as a matrix equation and find all solutions using Gaussian elimination:

$$x + 2y + 4z = 0, -x + 3y + z = -5, 2x + y + 5z = 3.$$

2. (a) Suppose  $T$  is a linear transformation represented by a matrix  $A$ . What does it mean for a vector  $v$  to be in the kernel of  $T$  (or, equivalently, in the kernel of the matrix  $A$ )?

(b) Let  $A$  be the matrix  $\begin{pmatrix} 1 & 2 & 5 \\ -2 & 0 & -2 \\ 3 & -1 & 1 \end{pmatrix}$ . Is  $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$  an element of the kernel of  $A$ ?

Why or why not?

3. Let  $V \subseteq \mathbb{R}^n$  be a subspace of  $\mathbb{R}^n$ . Define what it means for a list  $v_1, \dots, v_k$  of vectors in  $\mathbb{R}^n$  to be a) linearly independent, b) to span  $V$ , and c) to be a basis of  $V$ .

Let

$$A = \begin{pmatrix} 1 & 2 & 3 & -1 \\ -1 & 0 & 1 & -1 \\ -1 & 4 & 3 & -5 \end{pmatrix}.$$

Give a list of linearly independent vectors that span  $\ker(A)$ .

4. Let  $A$  be a  $n$  by  $m$  matrix, so  $A$  gives a linear transformation from  $\mathbb{R}^m$  to  $\mathbb{R}^n$ . Let  $x_1, x_2 \in \mathbb{R}^m$ . Assume that  $A(x_1) = A(x_2)$ . Show that  $x_1 - x_2$  is in the kernel of  $A$ . What property of matrix multiplication did you use in the process?

5. Let  $L$  be a line through the origin and let  $u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$  be a vector of length 1 lying along that line. Let  $A$  be a matrix whose effect on the plane is to reflect about the line  $L$ . Let  $v = \begin{pmatrix} -u_2 \\ u_1 \end{pmatrix}$ . In terms of  $u$  and  $v$  what is  $A(u)$ ? what is  $A(v)$ ?

Write  $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  as a linear combination of  $u$  and  $v$ . Use the answer to the previous question to compute  $A(e_1)$ .

6. Let

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 2 & 1 & -1 \end{pmatrix}.$$

(a) Determine whether the columns of  $A$  are linearly independent or not. Show your work.

(a) Use the row reduction method to find the inverse matrix  $A^{-1}$ .

(b) By using  $A^{-1}$ , solve the system

$$A\vec{x} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

for  $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ .

7. Compute the product  $AB$  of the two matrices  $A, B$  given below, if possible. If it is not possible say why it is not possible.

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \\ 3 & -2 \end{pmatrix}$$

$$B = \begin{pmatrix} -1 & 0 \\ 4 & 8 \end{pmatrix}$$

The product matrix  $AB$  represents a linear transformation from  $\mathbb{R}^m$  to  $\mathbb{R}^n$ . Determine  $m$  and  $n$ .

8. Find a basis of the subspace of  $\mathbb{R}^3$  defined by  $3x - y + z = 0$ . What is the dimension of this subspace?
9. Consider the matrix

$$A = \begin{pmatrix} 1 & 0 & 4 \\ 2 & 1 & 10 \\ 3 & 1 & c \end{pmatrix}.$$

- a) Find all the value(s) of  $c$  for which the matrix  $A$  is not invertible.
- b) For what value(s) of  $c$  does the linear system

$$A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 14 \\ 20 \end{pmatrix}$$

have no solution?

10. Suppose  $A$  and  $B$  are  $n \times n$  matrices such that  $AB = \mathbf{0}$  where  $\mathbf{0}$  is the  $n \times n$  all-zeros matrix. Show that if  $A$  is invertible, then  $B = \mathbf{0}$ .