

UMASS AMHERST
MATH 113
MATH FOR ELEMENTARY SCHOOL TEACHERS

SAMPLE FINAL QUESTIONS

1. List, in order, the steps in George Pólya's Approach to Problem Solving.
2. List five problem-solving strategies.
3. i) Convert 234_{six} to base ten numerals.
ii) Convert 312_{ten} to base three numerals.
4. i) Does the set of odd integers $\{\dots, -7, -5, -3, -1, 1, 3, 5, 7, \dots\}$ have the property of closure under multiplication? EXPLAIN.
ii) Does the set of even integers $\{\dots, -6, -4, -2, 0, 2, 4, 6, \dots\}$ have the property of closure under subtraction? EXPLAIN.
5. Mark the statement as True or False:
..... **In base two**, every whole number which ends with a 0, meaning the digit all the way on the right is 0, must be even.
6. Use the number line model to illustrate the following addition problem (showing how you obtain the answer): $-5 + 4 = ?$
7. Name the property of whole numbers being illustrated by each example: (e.g. $5+3=3+5$ illustrates the commutativity of addition)
..... i) $81 \times 3 = 3 \times 81$
..... ii) $5 \times (3 + 7 + 2) = 5 \times 3 + 5 \times 7 + 5 \times 2$
..... iii) $9 + (4 + 4) = (9 + 4) + 4$
..... iv) $1 \times 137 = 137$
8. Use the estimation technique of Clustering to find estimates for
i) $47+49+52+46+52+53+48$
ii) $9 \times 11 \times 13 \times 8 \times 8 \times 12 \times 11$
9. Using the range technique of estimation, determine lower and upper bounds for the following product: 10.95×19.37 .
10. Use the expanded algorithm for multiplication to find $87 \times 36 = ?$
11. When you give your students the subtraction problem $512 - 304$, one of them presents you with the following solution: $512 - 304 = 500 + 10 + 2 - 300 + 4 = 500 - 300 + 10 + 2 + 4 = 200 + 16 = 216$. Is his work correct? If so, explain why it is correct. If not, pinpoint the mistake(s).
12. An elementary school student presents you with the following remark: $1 + 2 + 3 = 6$, 6 divided by 6 is a whole number. $3 + 4 + 5 = 12$, 12 divided by 6 is a whole number. $7 + 8 + 9 = 24$, 24 divided by 6 is a whole number. $9 + 10 + 11 = 30$, 30 divided by 6 is a whole number. I think whenever you add three consecutive whole numbers, then divide the result by 6, you get a whole number.
i) What kind of reasoning is the student using?
ii) Is the conclusion supported by her evidence?
iii) Do you think the conclusion is correct? Explain why or why not.

13. Recalling that every division equation can be written as a multiplication equation, explain why $0 \div 0$ is undetermined.

14. Carry out the division $817 \div 4$ by the standard algorithm.

15. Fill in the blank: If a and b are integers, then a divides b means that

16. True or False:

T or F: The only infinite subset of the set of whole numbers is the set of whole numbers itself.

T or F: 5 divides 0

T or F: 0 divides 5

T or F: If a and b are positive integers, then $GCF(a, b) \times LCM(a, b) = ab$.

T or F: Mathematicians suspect that there are infinitely many prime numbers, but they are not able to prove for certain whether this is the case or not.

T or F: If a, b, c are integers such that a divides b and b divides c , then a divides c .

T or F: a whole number is divisible by 3 if and only if the sum of its digits is divisible by 3.

T or F: If a/b and c/d are rational numbers which are not equal, then $ad-bc$ is not 0.

T or F: One way to find 20% of a quantity is to multiply that quantity by $1/5$.

For the following problems, make sure you **SHOW ALL YOUR WORK**.

17. Determine whether 221 is a prime or not.

18. Find the prime factorization of 2520 (do the factor tree).

19. Find all factors of 154.

20. Determine $GCF(45, 420)$.

21. Determine $LCM(45, 420)$.

22. Use the charge model for integer subtraction to find the difference $4 - (-6)$. Do NOT convert this $4 + 6$ first; the answer is the same of course, but the modeling is different.

23. Use FRACTION BARS to show with diagrams and explain in words how to find the sum of one-fourth and two-thirds.

24. Without using decimals, place the following in numerical order from the smallest to the biggest: three-fifths, one-third, three-halves, two-thirds, five-fourths.

25. Give three rational numbers (as a fraction or as a decimal) which are simultaneously bigger than $7/4$ and less than $17/9$.

26. Use an area model to illustrate the computation of four-fifths times two-thirds, showing what the answer is.

27. Use an area model to illustrate the computation of three-fourths divided by three-eighths, showing what the answer is.