Math 233 **Practice Problems for Final Exam** **Fall 2006**

1st set of problems

1a) Express the double integral

$$\iint_R x^2y - x \ dA$$

as an interated integral and evaluate it, where R is the first quadrant region enclosed by the curves $y = 0$, $y = x^2$ and $y = 2 - x$. **b)** Find an equivalent iterated integral expression for the double integral in **1a)**, where the order of intergration is reversed from the order used in part **1a)**. (Do **not** evaluate this integral.)

2) Calculate the line integral

$$\int_C \mathbf{F} \cdot \mathbf{dr},$$

where $\mathbf{F}(x, y) = y^2x\mathbf{i} + xy\mathbf{j}$, and C is the path starting at $(1, 2)$, moving along a line segment to $(3, 0)$ and then moving along a second line segment to $(0, 1)$.

3) Evaluate the integral

$$\iint_R y\sqrt{x^2 + y^2}dA$$

with R the region $\{(x, y) : 1 \leq x^2 + y^2 \leq 2, \quad 0 \leq y \leq x.\}$

4a) Show that the vector field

$$\mathbf{F}(x, y) = \left\langle \frac{1}{y} + 2x, -\frac{x}{y^2} + 1 \right\rangle$$

is conservative by finding a potential function $f(x, y)$.
4b) Let C be the path described by the parametric curve $\mathbf{r}(t) =< 1 + 2t, 1 + t^2 >$ for $0 \leq t \leq 1$. Use your answer from **4a)** to determine the value of the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

5a) Find the equation of the tangent plane at the point $P = (1, 1, -1)$ in the level surface $f(x, y, z) = 3x^2 + xyz + z^3 = 1$.
b) Find the directional derivative of the function $f(x, y, z)$ at $P = (1, 1, -1)$ in the direction of the tangent vector to the space curve $\mathbf{r}(t) = \langle 2t^2 - t, t^{-2}, t^2 - 2t^3 \rangle$ at $t = 1$.

6) Find the absolute maxima and minima of the function

$$f(x, y) = x^2 - 2xy + 2y^2 - 2y$$

in the region bounded by the lines $x = 0$, $y = 0$ and $x + y = 7$.

2nd set of problems

1. Consider the function $f(x, y) = xe^{xy}$. Let P be the point $(1, 0)$.
(a) Find the rate of change of the function f at the point P in the direction of the point $(3, 2)$.
(b) Give a direction in terms of a unit vector (there are two possibilities) for which the rate of change of f at P in that direction is zero.

2. (a) Find the work done by the vector field $\mathbf{F}(x, y) = \langle x - y,\, x\rangle$ over the circle $\mathbf{r}(t) = \langle \cos t,\, \sin t\rangle$, $0 \le t \le 2\pi$.
(b) Use Green's Theorem to calculate the line integral $\int_C (-y^2)dx + xydy$, over the positively (counterclockwise) oriented closed curve defined by $x = 1\,, y = 1$ and the coordinate axes.

3. (a) Show that the vector field $\mathbf{F}(x, y) = \langle x^2y,\, \frac{1}{3}x^3\rangle$ is conservative and find a function f such that $\mathbf{F} = \nabla f$.
(b) Using the result in part (a) calculate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, along the curve C which is the arc of $y = x^4$ from $(0, 0)$ to $(2, 16)$.

4. Consider the surface $x^2 + y^2 - \frac{1}{4}z^2 = 0$ and the point $P(1,\, 2,\, -2\sqrt{5})$ which lies on the surface.
(a) Find the equation of the tangent plane to the surface at the point P.
(b) Find the equation of the normal line to the surface at the point P.

5. A flat circular plate has the shape of the region $x^2 + y^2 \le 1$. The plate (including the boundary $x^2 + y^2 = 1$) is heated so that the temperature at any point (x, y) on the plate is given by

$$T(x, y) = x^2 + 2y^2 - x$$

Find the temperatures at the hottest and the coldest points on the plate, including the boundary $x^2 + y^2 = 1$.

6. The acceleration of a particle at any time t is given by

$$\mathbf{a}(t) = \langle -3\cos t,\, -3\sin t,\, 2\rangle\,,$$

while its initial velocity is $v(0) = \langle 0,\, 3,\, 0\rangle$. At what times, if any, are the velocity and the acceleration of the particle orthogonal?

7. Find parametric equations for the line in which the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$ intersect.

3rd set of problems

1. a) Find the equation of the plane containing the points $P(1, 3, 0)$, $Q(2, -1, 2)$ and $R(0, 0, 1)$.

b) Find all points of intersection of the parametric curve $\mathbf{r}(t) = \langle 2t^2 - 2, t, 1 - t - t^2 \rangle$ and the plane $x + y + z = 3$.

2. Find the absolute maximum and minimum of the function $f(x, y) = x^2 + 2y^2 - 2y$ on the closed disc $x^2 + y^2 \leq 5$ of radius $\sqrt{5}$.

3. Evaluate

$$\iint_R xy \, dA$$

where R is the region in the first quadrant bounded by the line $y = 2x$ and the parabola $y = x^2$.

4. Consider the vector field $\mathbf{F}(x, y) = \langle 2xy + \sin y, x^2 + x \cos y + 1 \rangle$.

a) Show that $\mathbf{F}(x, y)$ is conservative by finding a potential function $f(x, y)$ for $\mathbf{F}(x, y)$.
b) Use your answer to (a) to evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where C is the arc of the parabola $y = x^2$ going from $(0, 0)$ to $(2, 4)$.

5. Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where $\mathbf{F} = \langle y^2 + \sin x, xy \rangle$ and C is the unit circle oriented counterclockwise.

6. Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where $\mathbf{F} = \langle y^2, 2xy + x \rangle$ and C is the curve starting at $(0, 0)$, travelling along a line segment to $(2, 1)$ and then travelling along a second line segment to $(0, 3)$.