

Practice Problems for Final Exam — Answers

1st set.

3.

$$\int \int_R y \sqrt{x^2 + y^2} dA = \int_0^{\pi/4} \int_1^{\sqrt{2}} r \sin \theta \sqrt{r^2} r dr d\theta = \left(-\cos \theta \Big|_0^{\pi/4} \right) \left(\frac{r^4}{4} \Big|_1^{\sqrt{2}} \right) = \frac{3}{8}(2 - \sqrt{2}).$$

4 a). $f(x, y) = x/y + x^2 + y$.

b). $f(3, 2) - f(1, 1) = 19/2$.

5 a).

$$5(x - 1) - (y - 1) + 4(z + 1) = 0.$$

b).

$$D_{\vec{u}} f(1, 1, -1) = \nabla f(1, 1, -1) \cdot \vec{u} = \langle 5, -1, 4 \rangle \cdot \frac{1}{\sqrt{29}} \langle 3, -2, -4 \rangle = \frac{1}{\sqrt{29}}.$$

6. $Max = 84$ at $(7, 0)$ and $Min = -1$ at $(1, 1)$.

2nd set.

1 a).

$$D_{\vec{u}} f(1, 0) = \langle 1, 1 \rangle \cdot \frac{1}{\sqrt{8}} \langle 2, 2 \rangle = \frac{4}{\sqrt{8}}.$$

b). $\vec{u} = \langle \pm \frac{1}{\sqrt{2}}, \mp \frac{1}{\sqrt{2}} \rangle$.

2 a). 2π .

b). $3/2$.

3 a). $f(x, y) = \frac{1}{3}x^3y$.

b). $f(2, 16) - f(0, 0) = 128/3$.

4 a). $2(x - 1) + 4(y - 2) + \sqrt{5}(z + 2\sqrt{5}) = 0$.

b).

$$\mathbf{r}(t) = \langle 1, 2, -2\sqrt{5} \rangle + t \cdot \langle 2, 4, \sqrt{5} \rangle$$

6. $t = 0$.

7. The line is given by

$$\mathbf{r}(t) = \langle 3, -1, 0 \rangle + t \langle 14, 2, 15 \rangle .$$

3rd set.

1 a). $2(x - 1) - 3(y - 3) - 7z = 0$.

b) $(6, 2, -5)$ when $t = 2$ and $(6, -2, -1)$ when $t = -2$.

2. $Max = 10 + 2\sqrt{5}$ and $Min = -1/2$.

3. $\left. \frac{x^4}{2} - \frac{x^6}{12} \right|_0^2 \simeq 2.67$.

4 a). $f(x, y) = x^2y + x \sin y + y$.

b). $f(2, 4) - f(0, 0) = 20 + 2 \sin 4$.

5. 0.

6. 3.