Practice Problems for Exam 1

1 $(-10, -7, 3)$.

2(a) $(1, 0, 4); t = 1, s = 2$.
(b) $x = 1 + u, y = -u, z = 4 + 2u$ and $x = 1 - v, y = v, z = 4 + 4v$.

3 $(4, 6, -6)$.

4(a) $< -1/2, -1/2, 1 >$
(b) $3\sqrt{3}/2$.
(c) $9/2$

5 $x = -3t, y = t, z = 5t + 1$.

6 L_1, L_2 intersect at $(0, 2, 1)$.

7(a) 13.
(b) $-7x + 6y - 8z = 1$.
(c) $\cos^{-1}(0.53)$.

8(a) $\mathbf{r}(4) = \langle 15, \frac{47}{3}, 8\rangle$.
(b) $\langle 15, \frac{47}{3}, 8\rangle + s\langle 8, 4, 1\rangle$.
(c) Yes, $\mathbf{r}(9) = P$.
(d) 4.

9 The x traces and y traces are hyperbolas, the z traces are ellipses.

10 $z = 4(x - 1)$.

11 $\frac{5\sqrt{6}}{12}$.

12 It is a hyperboloid of one sheet, $x^2 + y^2 - \frac{z^2}{2^2} = 1$.

13a) $< 0, -9.8 >$.
b) $50 < \sqrt{3}, 1 >$
c) $< 0, 5 >$
d) $t = 10.303$.
e) 892.266 m.

14 If (x, y) approaches $(0, 0)$ along y-axis, the limit of $f(x, y)$ is 0, while if (x, y) approaches $(0, 0)$ along the line $y = x$, the limit of $f(x, y)$ is 1, which is different from 0. Hence the limit of $f(x, y)$ does not exist as (x, y) approaches $(0, 0)$.

15 $2x + 4y - z = 6$.

16(a) $f_x = 3x^2 - y^2$, $f_y = -2xy + 1$, $f_{xx} = 6x$, $f_{yy} = -2x$, $f_{xy} = -2y$.
(b) $f_x = \frac{1}{x+\sqrt{x^2+y^2}}(1 + \frac{x}{\sqrt{x^2+y^2}})$

$f_y = \frac{1}{x+\sqrt{x^2+y^2}} \frac{y}{\sqrt{x^2+y^2}}$

$f_{xx} = -\frac{1}{(x+\sqrt{x^2+y^2})^2}(1 + \frac{x}{\sqrt{x^2+y^2}})^2 + \frac{1}{x+\sqrt{x^2+y^2}} \frac{\sqrt{x^2+y^2} - \frac{x^2}{\sqrt{x^2+y^2}}}{x^2+y^2}$

$f_{yy} = -\frac{1}{(x+\sqrt{x^2+y^2})^2}(\frac{y}{\sqrt{x^2+y^2}})^2 + \frac{1}{x+\sqrt{x^2+y^2}} \frac{\sqrt{x^2+y^2} - \frac{y^2}{\sqrt{x^2+y^2}}}{x^2+y^2}$

$f_{xy} = -\frac{\frac{y}{\sqrt{x^2+y^2}}}{(x+\sqrt{x^2+y^2})^2}(1+\frac{x}{\sqrt{x^2+y^2}}) + \frac{1}{x+\sqrt{x^2+y^2}}(-\frac{xy}{(\sqrt{x^2+y^2})^3})$.

17 $1.1e$.

18 $\mathbf{r}(t) =< t, t^2, 2t^2 + t^4 >$.