1. (a) Let $\mathbf{v} = \overrightarrow{AB} = \langle 2, 0, 1\rangle - \langle -1, 1, -1\rangle = \langle 3, -1, 2\rangle$. Since $A(2, 0, 1)$ lies on the line, then:

$$x = 2 + 3t,$$

$$y = 0 - t = -t,$$

$$z = 1 + 2t.$$

(b) Vector part of line l_1 is $v_1 = \langle 2, 3, -1\rangle$ and for line l_2 is $\mathbf{v}_2 = \langle 1, 1, 3\rangle$. Clearly, v_1 is not a scalar multiple of $\mathbf{v}_2$ and so these lines are not parallel. If these lines intersect, then for some values of t and s:

$$x = 1 + 2t = -1 + s \Rightarrow 2t = -2 + s,$$

$$y = 3t = 4 + s \Rightarrow 3t = 4 + s.$$

Solving these two linear equations yields:

$$t = 6 \text{ and } s = 14.$$

Plugging these values into $z = 2 - t = 1 + 3s$ yields the inequality $-4 \neq 43$, which means there is no solution and the lines do not intersect. Thus, the lines are *skew*.

2. (a) Consider the vectors $\overrightarrow{PQ} = \langle 2, -4, 0\rangle$ and $\overrightarrow{PR} = \langle 2, -1, -2\rangle$ which lie parallel to the plane. Then consider the normal vector:

$$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} i & j & k \\ 2 & -4 & 0 \\ 2 & -1 & -2 \end{vmatrix} = 8\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}.$$

So, the equation of the plane is given by:

$$\langle 8, 4, 6\rangle \cdot \langle x + 1, y - 2, z - 1\rangle = 8(x + 1) + 4(y - 2) + 6(z - 1) = 0.$$

(b) The normal to the plane is $\mathbf{n} = \langle 2, 1, -2\rangle$ and the point $P = (0, 1, 0)$ lies on this plane. Consider the vector from P to $(1, 2, -1)$ which is $\mathbf{v} = \langle 1, 1, -1\rangle$. The distance from $(1, 2, -1)$ to the plane is equal to:

$$|\text{comp}_{\mathbf{n}}| = \left|\mathbf{v} \cdot \frac{\mathbf{n}}{|\mathbf{n}|}\right| = \frac{5}{3}.$$

3. (a) After taking derivatives, we obtain:

$$\mathbf{r}_1'(t) = \langle -\sin(t-1), 2t, 4t^3 \rangle,$$

$$\mathbf{r}_2'(s) = \langle \frac{1}{s}, 2s-2, 2s \rangle.$$

At the point $(1,0,1)$, $t=1$ and $s=1$ and so, $\mathbf{r}_1'(1) = \langle 0,2,4 \rangle$ and $\mathbf{r}_2'(1) = \langle 1,0,2 \rangle$ are the related tangent vectors. Thus,

$$\cos(\theta) = \frac{\mathbf{r}_1'(1)}{|\mathbf{r}_1'(1)|} \cdot \frac{\mathbf{r}_2'(1)}{|\mathbf{r}_2'(1)|} = \frac{8}{\sqrt{20}\sqrt{5}}.$$

(b) Since $\mathbf{r}'(t) = \langle \sin(t), \cos(2t), e^t \rangle$, then $\mathbf{r}(t) = \int^t \mathbf{v}(s)ds$. Thus, $\mathbf{r}(t) = \langle -\cos(t) + x_0, \frac{1}{2}\sin(2t) + y_0, e^t + z_0 \rangle$ with $\mathbf{r}(0) = \langle -1 + x_0, y_0, 1 + z_0 \rangle = \langle 1,2,0 \rangle$. Thus, $x_0 = 2, y_0 = 2, z_0 = -1$ and so, $\mathbf{r}(t) = \langle -\cos(t) + 2, \frac{1}{2}\sin(2t) + 2, e^t - 1 \rangle$.

4. (a) For $f(x,y) = e^{x^2-y} + x\sqrt{4-y^2}$, we obtain:

$$f_x = 2xe^{x^2-y} + \sqrt{4-y^2},$$

$$f_y = -e^{x^2-y} + \frac{-xy}{\sqrt{4-y^2}},$$

$$f_{xy} = -2xe^{x^2-y} + \frac{-y}{\sqrt{4-y^2}}$$

(b) For function $f(x,y) = \sin(2x+y)+1$, $f_x = 2\cos(2x+y)$, $f_y = \cos(2x+y)$. At the point $(0,0,1)$ on the graph, we have: $f_x(0,0,1) = 1$, $f_y(0,0,1) = 1$. The equation of the tangent plane is $z = f(0,0,1) + f_x(0,0,1)(x-0) + f_y(0,0,1)(y-0) = 1+x+y$.

5. (a) Since $g(x,y) = ye^x$, then $g_x = ye^x$ and $g_y = e^x$. The linear approximation L to $g(x,y)$ at $(x,y) = (0,2)$ is given by:

$$L(0+\Delta x, 2+\Delta y) = g(0,2) + g_x(0,2)\Delta x + g_y(0,2)\Delta y = 2 + 2\Delta x + \Delta y.$$

For this problem $\Delta x = .1$ and $\Delta y = -.1$. So, the linear approximation gives: $2 + .2 - .1 = 2.1$.

(b) Complete squares to obtain from $x^2 + y^2 + z^2 + 6z = 16$, the equation:

$$x^2 + y^2 + (z+3)^2 = 16 + 9 = 25.$$

Hence, the center is at $C = (0,0,-3)$ and the radius is 5.

(c) For the graph $z = f(x,y) = \sqrt{16 - x^2 - y^2}$, the contour map of the level curve for $k = 2$ is formed by setting: $2 = \sqrt{16 - x^2 - y^2}$ or $4 = 16 - x^2 - y^2$ which yields the circle $x^2 + y^2 = 12$, centered at the origin with radius $\sqrt{12}$. Similarly one obtains contour circles for the other values of k. One then draws these circles in the xy-plane and labels them according to their $z = k$ values.