Solutions of Problem #2, Midterm Exam 2, Calculus, Math 233

Given the function $f(x, y) = x^2y + ye^{xy}$:

(a) Find the linearization of f at the point $(0, 5)$ and use it to approximate the value of f at the point $(0.1, 4.9)$.

Solution of part **(a)**:

Linearization of f at $(0, 5)$ is equal to

$$L(x, y) = f(0, 5) + f_x(0, 5)(x - 0) + f_y(0, 5)(y - 5).$$

Since the first partial derivatives of $f(x, y)$ are

$$f_x(x, y) = 2xy + y^2e^{xy}, \quad f_y(x, y) = x^2 + e^{xy} + xye^{xy},$$

we have

$$L(x, y) = 5 + 25 \cdot (x - 0) + 1 \cdot (y - 5) = 25x + y.$$

Then the approximate value of f at $(0.1, 4.9)$ is

$$L(0.1, 4.9) = 25 \cdot 0.1 + 4.9 = 7.4$$

Grading of part **(a)***:*

-2 points if the value of $L(0, 5)$ was not found, or the term e^{xy} or xye^{xy} was missed in $f_y(x, y)$ (which also follows to **-2** points in parts (b) and (c)).
-3 points if $L(x, y)$ found wrong, but there is a general formula for it, or derivatives were computed while the function $L(x, y)$ not.

(b) Suppose that $x(r, \theta) = r \cos\theta$ and $y(r, \theta) = r \sin\theta$. Calculate f_θ at $r = 5$ and $\theta = \frac{\pi}{2}$.

Solution of part **(b)***:*

Using the Chain Rule,

$$f_\theta = f_x \cdot x_\theta + f_y \cdot y_\theta = (2xy + y^2e^{xy}) \cdot (-r \sin\theta) + (x^2 + e^{xy} + xye^{xy}) \cdot (r \cos\theta)$$

At $(r, \theta) = (5, \frac{\pi}{2})$ we have $(x, y) = (0, 5)$, $(x_\theta, y_\theta) = (-5, 0)$ and

$$f_\theta(5, \pi/2) = 25 \cdot (-5) + 1 \cdot 0 = -125.$$

Grading of part **(b)***:*

-3 if the solution involves direct computation of partial derivative of $f(r, \theta)$, but follows to the mess

(c) Suppose a particle travels along a path $(x(t), y(t))$, and that $F(t) = f(x(t), y(t))$ where $f(x, y) = x^2y + ye^{xy}$. Calculate $F'(3)$, assuming that at time $t = 3$ the particle's position is $(x(3), y(3)) = (0, 5)$ and its velocity is $(x'(3), y'(3)) = (3, -2)$.

Solution of part **(c)***:*
By the Chain Rule

$$F'(t) = f_x \cdot x'(t) + f_y \cdot y'(t),$$

and when $t = 3$, we have

$$F'(3) = 25 \cdot 3 + 1 \cdot (-2) = 75 - 2 = 73.$$

Grading of part **(c)***:*

-3 if in the Chain Rule instead of the values of partial derivatives were the coordinates of the point, i.e. 0 and 5 instead of 25 and 1 respectively.

Each part of the problem #2 cost **-4** to **-5** points if most part of the problem was solved not correct. Also, **-1** for the general mess/or no comprehensive explanation/minor computational mistake, and **-2** if there are no numerical results.