DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS

MATH 233 **FINAL EXAM** **Spring 2007**

NAME:________________________________ Student ID# ______________________

Section Number:______________________ Instructor's Name: ______________________

In problems that require reasoning or algebraic calculation, it is not sufficient just to write the answers. You must explain how you arrived at your answers, and show your algebraic calculations. Definite integrals must be solved symbolically, not by calculator.

You can leave answers in terms of fractions and square roots, but if approximate numerical answers are used, they should be rounded off to 4 significant figures.

1.	(18)	
2.	(12)	
3.	(10)	
4.	(15)	
5.	(15)	
6.	(15)	
7.	(15)	
Total		

Perfect Paper ⟶ 100 Points.

There are eight pages, including this one, in this exam and seven problems. Make sure you have all the pages before you begin!

(1) a) (8 points) Find an equation of the plane in $\mathbb{R}^3$ which contains the three points $(1,1,0)$, $(2,1,-3)$ and $(0,2,5)$.

b) Consider the surface $x + x^2 + y^2 - 2z^2 = 1$ and the point $P(1,1,1)$ which lies on the surface.

(i) (5 points) Find an equation of the tangent plane to the surface at P.

(ii) (5 points) Find an equation of the normal line to the surface at P.

(2) (12 points) Find the maximum and minimum values of the function

$$f(x, y) = x^2 + y^2 - 2x + y$$

on the disc $x^2 + y^2 \leq 5$.

(3) Consider the two dimensional triangle T in the xy-plane with vertices $(0, 0)$, $(2, 1)$ and $(0, 3)$. Consider the mass density function $\rho(x, y) = 2x^2 + 1$ on T.

a) (6 points) What is the total mass m of the triangle T?

b) (4 points) What is the x-coordinate $\overline{x}$ of the center of mass $(\overline{x}, \overline{y})$ of T with respect to this density function ρ? Write down the answer using as part of the answer an integral but do not evaluate the integral!

$$\overline{x} =$$

(4) (15 points) Find the volume of the solid under the surface $z = 1 - x^2 - y^2$ and above the xy-plane.

(5) (15 points) Determine whether the following vector fields are conservative or not. Find a potential function for those which are indeed conservative.

(a) $\mathbf{F}(x, y) = (x^2 + e^x + xy)\mathbf{i} + (xy - \sin(y))\mathbf{j}$.

(b) $\mathbf{F}(x, y) = (3x^2y + e^x + y^2)\mathbf{i} + (x^3 + 2xy + 3y^2)\mathbf{j}$.

(6) (15 points) Evaluate the line integral

$$\int_C (x^2 + y)\,dx + (xy + 1)\,dy + z\,dz$$

where C is the curve starting at $(0,0,0)$, traveling along a line segment to $(1,0,0)$ and then traveling along a second line segment to $(1,2,1)$.

(7) a) (10 points) Use Green's Theorem to show that if $D \subset \mathbb{R}^2$ is the bounded region with boundary a positively oriented simple closed curve C, then the area of D can be calculated by the formula:

$$\text{Area} = \frac{1}{2}\int_C -y\,dx + x\,dy$$

b) (5 points) Consider the ellipse $4x^2 + y^2 = 1$. Use the above area formula to calculate the area of the region $D \subset \mathbb{R}^2$ with boundary this ellipse. (Hint: This ellipse can be parametrized by $r(t) = \langle \frac{1}{2}\cos(t), \sin(t)\rangle$ for $0 \leq t \leq 2\pi$.)