

## Math 300— Homework Set 3

Due Thursday, February 28

1. Read chapter 1 in the book.
2. In chapter 1: problems 9, 10, 12
3. In chapter 1: problems 17, 18, 19, 20, 21
4. In chapter 1: problems 26, 27, 28.
5. In chapter 1: problems 55, 56, 58, 59, 60
6. In chapter 1: problems 65, 67, 68, 69
7. Consider the following properties of the real numbers  $\mathbb{R}$ :
  - (a) The commutative property of addition and multiplication
  - (b) The associative property of addition and multiplication
  - (c) The distributive law
  - (d) Each real number  $a$  has an additive inverse  $-a$ . This means that for all  $a \in \mathbb{R}$  there exists  $-a \in \mathbb{R}$  such that  $a + (-a) = 0$ .
  - (e)  $0 \cdot a = 0$
  - (f)  $1 \cdot a = a$
  - (g) For all  $c \in \mathbb{R}$ ,  $a > b \Rightarrow a + c > b + c$
  - (h)  $a > b$  and  $c > 0 \Rightarrow ac > bc$
  - (i)  $-a = (-1)a$  and  $-(-a) = a$ .
8. Using only the properties in #7, prove that the following are true.
  - (a) For  $a \in \mathbb{R}$ ,  $a > 0$  if and only if  $-a < 0$ . Hint: use #7(f).
  - (b) For  $a, b, c \in \mathbb{R}$ , if  $a > b$  and  $c < 0$ , then  $ac < bc$ . Hint: First rewrite  $a > b$  as  $a + (-b) > 0$  using one of the properties.
  - (c) For  $a, b \in \mathbb{R}$ ,  $a > b > 0 \Rightarrow a^2 > b^2 > 0$ .
  - (d) Prove a converse to the previous statement: namely, if  $a^2 > b^2$  and  $a$  and  $b$  are positive real numbers, then  $a > b$ .
  - (e) The inequality

$$\frac{a+b}{2} \geq \sqrt{ab}$$

holds for all nonnegative  $a, b \in \mathbb{R}$ . This is a useful inequality: it says that the geometric mean of two numbers is less than or equal to the arithmetic mean of the two numbers.