Math 300— Homework Set 1

Due Thursday, February 7

1. The material for the next few lectures comes from Sections 2.1, 2.2, and 2.5 in the book.

2. (For fun) Give a definition of each of the following mathematical terms from memory:

   - A circle
   - A function
   - The intersection of a set X and a set Y

3. Let a, b, c be three numbers. Suppose that a divides b, but a does not divide c. Prove that a does not divide $b + c$. (hint: assume that a does divide $b + c$ and derive a contradiction).

4. Use the previous problem to find a number that is not divisible by any of the numbers $2, 3, 4, 5, \ldots, 98, 99, 100$.

5. How many positive divisors does the number $2^{13}3^{2008}$ have? Answer the same question for any number of the form $p_1^{k_1}p_2^{k_2}$ where p_1 and p_2 are different primes and k_1 and k_2 are natural numbers.

6. From section 2 in the book: # 94.

7. Simplify the algebraic expression:

$$(a - 1)(1 + a + a^2 + \cdots + a^n).$$

8. Use the previous problem to show that $1 + 2 + 4 + 8 + \cdots + 2^n = 2^{n+1} - 1$.

9. Show that if m is not prime, then $2^m - 1$ is not prime (hint: factor m and then use a previous problem).

10. Prove or disprove: if p is prime, then $2^p - 1$ is prime.

11. Show that if $2^p - 1$ is prime, then $2^{p-1}(2^p - 1)$ is a perfect number. Indeed, every perfect number that is even must have this form. It is an open question whether there are any odd perfect numbers.