Math 300 – Homework Set 8

Due Thursday, May 8

1. Read section 6.9 in the book on permutations. The set of permutations on a set X is an example of a **group**. The multiplication in this group is composition of permutations. You will see more on groups in Math 411.
2. Give reasonable explanations for your answers to the following questions.
   (a) Let $f : \mathbb{P}_n \to \mathbb{P}_m$ where $m < n$ (book's notation). What can you say about f? This is called the pigeon-hole principle.
   (b) What if $m > n$?
   (c) If $n = m$ and f is injective, what can you say about f? What if $n = m$ and f is surjective?
   (d) What is the cardinality of the set of permutations of $\mathbb{P}_n$? In other words, what is the cardinality of the set of all bijections from $\{1, 2, \dots, n\}$ to itself?
3. # 101 in Chapter 6.
4. Show that the following two sets have the same cardinality as $\mathbb{R}$:
   (a) The open interval $(0, 1) = \{x \in \mathbb{R} \mid 0 < x < 1\}$. (Hint: find a well-known function that maps $(0, 1)$ to $\mathbb{R}$ bijectively)
   (b) The closed interval $[0, 1]$. (Hint: you may have to use a result from class that we did not prove).
5. Read the notes by Bill Meeks, from page 49 until page 60. We will be studying a little topology for the next five lectures. Our goals are modest: to define the notions of metric space and topological space and to explain in these contexts the concepts of continuous function, connectedness, and compactness.
6. On pages 64-66, do problems 1-6, 8, 10, 17.