Final Exam Practice Sheet, Math 131, Spring 2008

1. Let $f(x) = x^3 - 2$. Explain, using the Intermediate Value Theorem, why there is a root of $f(x)$ between $x = 1$ and $x = 2$.

2. Continuing with the previous problem, explain, using Rolle's Theorem (which is a special case of the Mean Value Theorem), why there can not be more than one root of $f(x)$ between $x = 1$ and $x = 2$.

3. Continuing with the previous problem, estimate this root by using Newton's Method with $x_1 = 1$. Do a single iteration and find x_2 (do this without using a calculator). Now set up the equation for x_3. Evaluate x_3 using your bare hands or your calculator.

4. Continuing with the previous problem, estimate this root by using linear approximation for the function $g(x) = \sqrt[3]{x}$ with $a = 1$. In other words, estimate $\sqrt[3]{2}$ using linear approximation.

5. Suppose a differentiable function satisfies $f(0) = 1$ and $1 \leq f'(x) \leq 2$ whenever $0 \leq x \leq 3$. Explain why $4 \leq f(3) \leq 7$. (Hint: see Section 4.2).

6. Practice antiderivatives: # 3, 11, 15, 31, 49 in Section 4.9

7. Interpret definite integrals in terms of areas of triangles, rectangles, and circles: # 33, 34 in Section 5.2

8. Estimate definite integrals by dividing the interval from a to b into n equal pieces:

   (a) $\int_1^5 \cos(x^2)\,dx$, with $n = 2$ and then again with $n = 4$. Use left endpoints. Set up the sum, but do not evaluate it.

   (b) $\int_{-1}^{2} \frac{1}{x+2}\,dx$, with $n = 3$. Use right endpoints. Set up the sum and this time evaluate it.

9. Practice optimization: # 3, 17, 19, 24 in Section 4.7

10. Practice L'Hopital's Rule by doing some of the suggested problems from Section 4.4 (see the webpage).

11. Practice suggested problems from Section 4.1 and Section 4.3. In a sense, these are the two most important sections of the course. See also homeworks 10 and 11.

12. In addition to the above highlighted topics, make sure you can:

    (a) take the derivative of any reasonable function by using the product rule, quotient rule, or chain rule and by knowing the derivatives of all the basic functions we have studied. (various sections in Chapter 3; see Exam 2 and old homeworks)

(b) do implicit differentiation and related rates (Sections 3.5, 3.6, and 3.9)

(c) find the equation of the tangent line to a curve at a given point. (various sections in Chapters 2 and 3).

(d) find vertical and horizontal asymptotes (Sections 2.2 and 2.6)

(e) determine where a function is continuous and where it is differentiable (Sections 2.5 and 2.8)

(f) use the Intermediate Value Theorem, Rolle's theorem, and the Mean Value Theorem (Sections 2.5 and 4.2)

(g) find the absolute maximum and minimum of a differentiable function on a closed interval $[a, b]$ (Section 4.1).

(h) use approximation methods (Linear approximation, Section 3.10; and Newton's Method, Section 4.8).