

Primitive ideals of down-up algebras.

Down-up algebras were introduced by Benkart and Roby in 1998 from combinatorial considerations. They can be viewed as close relatives of enveloping algebras of low-dimensional Lie algebras: for example, down-up algebras have Verma modules, and their representation theory has similarities with the classical representation theory of Lie algebras.

Because down-up algebras have low dimensions, it is possible to do explicit calculations with them. Unfortunately, the calculations tend to involve a lot of case-by-case considerations. Last year it struck me that this could actually be an advantage, especially at an undergraduate college. I invited a student to do a summer research project on this topic, thinking that the student could probably handle some of the special cases after seeing one of them done. We successfully calculated the primitive ideals of non-noetherian down-up algebras, giving a reasonably explicit list of their generators.

I will describe this work, starting with the definition of down-up algebras, a description of their Verma modules (which play a central role in classifying the primitive ideals), and probably ending with one of the cases we calculated. No heavy technical machinery will be used; we mostly deal with multivariable polynomials.