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Research Statement

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I. Math Resarch

My primary research interests and dissertation are in the field of large deviations, which comes under the broad heading of probability theory. It brings together topics from statistics, functional analysis, spectral theory, convex analysis and ergodic theory. Simply put, we say a sequence of random objects satisfies a large deviation principle (LDP) if we can find a function that models the asymptotic probability that the sequence is in some subset as time or space goes to infinity. In other words, an LDP provides the probability distribution of the sequence of random objects "in the long run." The name large deviations principle comes from the fact that it characterizes the behavior of the random sequence not just near its mean, but also at large deviations away from the mean. These largely deviant events can be of great importance by themselves (e.g. stock market crash, casino jackpots), but also by studying the unlikely events we can characterize the likely events.

A basic example of LDPs is the proportion of heads one gets when tossing a coin. If we let $\{S_n\}$ be the sequence of random variables that measures the proportion of heads for n fair coin tosses, then we know the law of large numbers tells us that this proportion converges to ½, but what about the actual probability distribution? Applying the LDP here will give us a form to find a rate function, I(x), which among other things describes:

$$\text{Prob}\{ \mid S_n - ½ \mid > x \} \approx \exp(-n\, I(x)), \text{ as } n \text{ grows to } \infty$$

Although the Central Limit Theorem could be applied here, it only measures fluctuations of order $1/n^{½}$, whereas the LDP will handle larger fluctuations of order 1. In addition, an LDP exists in more general settings as well. For example, the random object need not be an average of iid random variables, nor does it need to even be real-valued. An LDP exists for dependent random variables like Markov Chains, and the random objects can be measure-valued, for example.

Applications of large deviations can be found in economics, physics, statistics, and other fields. The one I focused on for my dissertation comes from statistical mechanics and deals with finding an LDP for a large particle system, which could be used to model a gas, for example. For the structure, we use a lattice (Z^d), and at each lattice site, there is a particle, with some "spin" (e.g. positive/negative charge or pointing up/down). So the space of all possible spin configurations on the lattice represents all the possible states that we could find our system in at any time. In other words, this is the sample space for our experiment. Now, we define an interaction as a function that describes how particles interact with each other. Associated to an interaction, we have two important objects. First, we get a Hamiltonian, which tells us how much energy (per unit of volume) is generated by the system for a fixed configuration of spins and will serve as the random variable for the experiment. According to how the particles interact, different configurations will produce different amounts of energy. Also, we get a Gibbs measure on the space of all possible spin configurations of the lattice, which tells us the expected distribution of spins across our system at equilibrium, according to this interaction. The Gibbs measure will play the role of the probability measure in our LDP. Without going into too many details, the result I proved was that using the Gibbs measure of one interaction, the Hamiltonian of another interaction satisfies an LDP, and the rate function that works can be interpreted as an entropy of the system. I was also able to prove this result in the quantum case. In the quantum version, the random objects are replaced by linear operators, and the probability measures are replaced by partial traces. This case is significantly harder due to the noncommutativities that arise in this setup. Although the classical version was known, it was not proved directly, so one did not get an appreciation or understanding of why it was true. The direct proof I designed is more transparent, and was also useful in proving the quantum version, a result which was not known.

I plan to continue my research in large deviations and other fields with current and, hopefully, future colleagues. In large deviations, a main series of problems comes from trying to understand and translate problems and results from classical mechanics to quantum mechanics. Although these analogs exist, the difficulty of the quantum setup makes them more challenging to formulate and demonstrate. In addition to these theoretical problems, I am interested in applying large deviation results in the study of actual systems. The LDP results in my dissertation were quite general, but applying them and other results to specific systems can provide a better understanding of how we expect them to behave. I've worked briefly in helping generate computer simulations that visually display these results based on numerical approximations, and this is an area that I'd also like to pursue. I would welcome the opportunity to work with colleagues from the department in other areas, and my broad background would allow me to do this. I also feel that my computer science and modeling skills would make me a valuable asset for department research in general.

II. Math Education Research

Aside from my math interests, education is something that I would like to continue to research. As an undergrad, I did extensive research to try and understand the causes, symptoms and treatments for math anxiety disorder. I conducted a survey of faculty, tutors and students to learn about their frustrations and suggestions with this very real problem. I discovered that a primary cause of this problem was the student's continued belief in so-called "math myths". Excuses like "I haven't done math in years, so I'm not going to get this" or "I was put in the math 'low-track' in school, so I can't do this" need to be dispelled by educators. This will force students to stop using them as a crutch, which is a danger with any learning disability. Research is needed in math education to tell us how to reform teaching methods, assessment methods and curricula to deal with this and other problems facing our students. The typical student who enters our classrooms next year will not be the same as the one who did ten years ago. Education research is the key to helping us reform methods to adapt to our ever-changing student body.

III. Undergrad Research

In addition to continuing with my own research, one of my top priorities and interests will be involving undergrads in research. As an undergrad, I did two research projects during my senior year. These experiences were invaluable to me because they forced me to think with more independence and innovation, instead of just following along with the syllabus of a course. These are skills that we all need, but I don't think we always get in college, which is why some graduating seniors might feel unprepared for the "real world". I also think that involvement in undergrad research increases one's interest and confidence in pursuing a graduate degree.

Within my field, there are ample opportunities for students to do research with me. Although more sophisticated math is needed to do rigorous proofs, applications of large deviations are easily understood. Markov chains are extremely useful in modeling the behavior of a variety of systems. Moreover, Markov chains satisfy a large deviation principle. An interesting research project might consist of developing a Markov model for a particular system, applying the large deviation principle, and discussing the meaning of the principle as it applies to the specific problem. That is, what is a large deviation for this problem and why do we care about it? In addition, computer simulations could be designed and ran to collect supporting data. I would love working with a student on a problem like this, and it would be pretty accessible, yet very challenging. I would also be happy to work with a student who had some specific problem in mind, or wanted to work in a totally different field.