CHAPTER 5

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Abstract. This is a summary of chapter 5 of Stewarts book. This note is not complete, one must read the book for examples.

1. Section 5.1,5.2

"To compute an area we approximate a region by rectangles and let the number of rectangles become large. The precise area is the limit of these sums of areas of rectangles."

- The area of a rectangle with sides a and $b = a.b$
- The area A of the region under the graph of the continuous function f on the interval $[a, b]$:

$$A = \lim_{n \longrightarrow \infty} \Delta x[f(x_1) + ... + f(x_n)],$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i.\Delta x$ for $0 \leq i \leq n$.
- The *definite integral of f from a to b*:

$$\int_a^b f(x)dx = lim_{n \to \infty} \Delta x[f(x_1^*) + ... + f(x_n^*)],$$

where $x_i^* \in [x_i, x_{i+1}]$ is a *sample point* for $0 \leq i \leq n-1$.
- $\int_a^b f(x)dx =$ area under the graph of f from a to b.
- $\int_a^a f(x)dx = 0$.
- $\int_a^b cdx = c(b-a)$ where c is a constant.
- $\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$.
- $\int_a^b cf(x)dx = c \int_a^b f(x)dx$ where c is a constant.
- $\int_b^a f(x)dx = -\int_a^b f(x)dx$.
- If $f(x) \geq 0$ then $\int_a^b f(x)dx \geq 0$.
- If $f(x) \geq g(x)$ then $\int_a^b f(x)dx \geq \int_a^b g(x)dx$.
- If $m \leq f(x) \leq M$ then $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$.
- *Midpoint rule*:

$$\int_a^b f(x)dx \approx \Delta x \sum_{i=1}^{n} f(\frac{x_{i-1} + x_i}{2}).$$

2. Section 5.3,5.4

- *Fundamental Theorem of Calculus(FTC), Part 1*: If f is a continuous function on $[a, b]$ and $g(x) = \int_a^x f(t)dt$, then g is continuous on $[a, b]$ and $g^{'}(x) = f(x)$ for all $x \in (a, b)$.
- FTC+Chain rule : If
$$g(x) = \int_a^{h(x)} f(t)dt,$$
then
$$g^{'}(x) = f(h(x)).h^{'}(x).$$
- F is an *antiderivative* of f if $F^{'}(x) = f(x)$. We write $\int f(x)dx = F(x)$.
- FTC, Part 2/Net Change Theorem :
$$\int_a^b F^{'}(x) = F(b) - F(a).$$

3. Section 5.5

- *The Substitution Rule*:
$$\int f(g(x))g^{'}(x)dx = \int f(u)du.$$
- $\int_a^b f(g(x))g^{'}(x)dx = \int_{g(a)}^{g(b)} f(u)du.$
- If $f(-x) = f(x)$(f *even function*), then $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx.$
- If $f(-x) = -f(x)$(f *odd function*), then $\int_{-a}^{a} f(x)dx = 0.$