

MATH. 461 – Fall 04
Homework Problem Due Monday 12/13

1) Consider the following polynomials in $\mathbf{C}[x, y, z, w]$:

$$f_1 = 3x^2 + 2yz - 2xw; \quad f_2 = 2xz - 2yw$$

$$f_3 = 2xy - 2z - 2zw; \quad f_4 = x^2 + y^2 + z^2 - 1$$

(a) Find a Gröbner basis for the ideal $I = \langle f_1, f_2, f_3, f_4 \rangle$. (Use your favorite monomial order)

(b) Use your result above to bound the number of complex solutions of the system of equations

$$f_1 = f_2 = f_3 = f_4 = 0$$

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(c) Find a Gröbner basis for the radical ideal \sqrt{I} .

(d) Use the previous result to obtain the exact number of distinct complex solutions for the system

$$f_1 = f_2 = f_3 = f_4 = 0$$

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