FACULTY PROFILE: NATHANIEL WHITAKER

In Professor Nathaniel Whitaker’s mind, he is an unlikely person to have pursued a career in mathematics. He is one of the rare African-Americans holding a PhD in math and – even rarer – a full professor at a major research university.

Nate grew up in southeastern Virginia during a time of racial segregation. In his early years, he remembers segregated stores, movie theaters, restaurants and restrooms. He learned from his parents how to navigate in that environment. His parents did not have many educational opportunities. His father completed the sixth grade, and his mother the ninth, but both had to quit school to work to help support their families. Their knowledge about the education process was limited but they knew of its importance.

In spite of their lack of education, Nate feels they were exceptional role models. He remembers his father leaving for work in the morning at 6am, working at a shipyard until 2pm, coming home and leaving for a second job from 4pm until midnight and starting again the next day. On the weekends his father also had a part time job.

WHY I WANT TO TEACH HIGH SCHOOL MATH

Mathematics is used constantly, often without our noticing it. It is the basis of problem solving and is a gateway science because it is fundamental to many other subjects including physics, engineering, computer science, chemistry, and biology. Therefore, it is crucial that young learners gain a solid understanding of mathematics before moving on to a higher level of learning. My goal is to become a high school mathematics teacher so that I may help our nation’s youth attain a profound insight into this subject.

In the K-12 classroom, the importance of mathematics is sometimes difficult for the students to realize. Many feel a disconnect between what they learn in the classroom and what they experience in the real world. Translating the curriculum into real-world occurrences is an important supplement to the material. In formulating a teaching philosophy, I have been greatly inspired by my high school physics teacher, who assigned experiments to complement his lectures. Using modern technology, we saw how the equations form from concrete

Article continues on page 25

IOANNIS PANTAZIS WINS DARPA AWARD

Ioannis Pantazis, a Postdoctoral Research Associate in our department, won a $50,000 award from the Defense Advanced Research Projects Agency (DARPA) as the bronze medal recipient in a predictive modeling contest. The contest focuses on the debilitating, mosquito-borne disease known as the chikungunya virus. The goal is to accurately forecast how the disease might spread in the Americas and the Caribbean. DARPA wants to find technologies that U.S. health officials can use to make decisions in the case of an outbreak. The announcement of the award points out that the participants, who include Ioannis, “identified gaps in current forecasting capabilities and created a set of tools that can immediately help improve forecasting and guide response decisions for the current chikungunya outbreak.” Details are given at http://www.darpa.mil/news-events/2015-05-27.

Article continues on page 28

Athena Polymeros ‘16 is majoring in mathematics with a concentration in teaching and a minor in Spanish.
NEW CHALLENGE PROBLEMS

Here is stringy sextet of challenging problems, drawn – and quartered – for your intellectual stimulation from the 2015 Jacob-Cohen-Killam prize exam:

**Problem 1.** Can one find elements $f_k$ and $f_{k+2}$ for some natural number $k$ in the Fibonacci sequence ($f_{n+1} = f_n + f_{n-1}$, $f_0 = 1$) which are both divisible by 2015?

**Problem 2.** Imagine a pair of nails located 10 cm apart at the same height on a vertical wall. How would you wind a long string (attached to a picture frame in the usual way) around those nails so that the picture is safely hanging on the wall but when removing either nail (but not both, of course) the picture will fall to the ground?

**Problem 3.** It is well known that the harmonic series $1 + 1/2 + 1/3 + ...$ is divergent. Will it still diverge if we remove all fractions which contain a digit 3 in the denominator?

**Problem 4.** What is the chance that exactly 2 people in a family of 3 were born on the same day of the week (assuming it’s equally likely to be born on any day of the week, and no multiple births)?

**Problem 5.** Five mathematicians – Alex, Franz, Jenia, Paul and Rob – sit around a table, each with a huge plate of cheese. Instead of eating it, every minute each of them simultaneously passes half of the cheese in front of him to his neighbor on the left and the other half to his neighbor on the right. Is it true that the amount of cheese on Franz’s plate will converge to some limit as time goes to infinity?

**Problem 6.** Cut a round disk into the greatest possible number of pieces using 6 straight lines.
DEPARTMENT HEAD’S MESSAGE

When I took on the job of Department Head a year ago, I didn't set out to make major changes in the Department. Instead, I decided to focus on the basics. In particular I put a special emphasis on making sure that we made the best hires possible. I'm extremely pleased to report that thanks to the hard work of two search committees, both of our tenure track positions were filled by our top candidates. Yao Li is a probability theorist coming to us from the Courant Institute of Mathematical Sciences at New York University. Yao is a 2012 PhD from Georgia Tech specializing in dynamical systems, stochastic processes, and their applications; some of his work in systems biology is in collaboration with neuroscientists. Patrick Flaherty joins our statistics group from the Department of Biomedical Engineering at Worcester Polytechnic Institute. Pat did his doctoral work in computer science at the University of California Berkeley, though one of his advisors was in the Department of Statistics there. He did his postdoctoral work at Stanford. Pat studies and develops statistical methods to analyze large-scale genomic data for the purpose of providing the best clinical diagnosis and treatment of cancer and other genetic diseases.

We also had a very successful year in hiring Visiting Assistant Professors. Joining us will be Stathis Charalampidis (applied math), Liubomir Chiriac (number theory), Ava Mauro (probability), Zheng Wei (statistics), and Gufang Zhao (representation theory/algebraic geometry). The support of CNS Dean Steve Goodwin in increasing the number of VAPs funded by the College from 5 to 8 is but one indication of his strong support for our department's mission in research as well as teaching.

Speaking of teaching, we have added two new courses to our regular roster of classes: Math 475 History of Mathematics returns to the curriculum after a long hiatus, and analysis now has a two-semester sequence thanks to the addition of Math 524 Analysis II. The first addition benefits all math majors but is particularly important for those who will go on to teach secondary mathematics. The second addition is especially useful for those who will go on to graduate study in mathematics. In the coming year, the statistics curriculum will be getting a fresh look, and we hope to bring in more new elective courses every year as part of a process contemplated by the Undergraduate Affairs Committee for updating the upper division curriculum. Our efforts in this direction are all the more important as the number of math majors is experiencing a sharp increase, from 475 a year ago to 583 this past spring. As part of the University's initiative to examine academic advising, we will be implementing some enhancements in advising in the coming year.

Among many honors and awards received by the faculty this year, I’d like to highlight just two here. Professor Andrea Nahmod joins Emeritus Professor Floyd Williams as Fellow of the American Mathematical Society. Professor Panos Kevrekidis is the first member of our department to be elected Fellow of the American Physical Society. Congratulations are also due to Daeyoung Kim and Alexei Oblomkov, who achieved tenure and promotion to Associate Professor, as well as to Siman Wong, who was promoted to the rank of Professor.

We had an extremely successful strategic planning process this year and produced a set of documents which will help guide the next steps as the University moves to a decentralized budget model over the next two years. Other major initiatives which will continue include drafting a plan for increasing the size of the faculty, improving the cluster computing infrastructure, expanding online offerings, and further work on space planning.

The support of our alumni and friends means a great deal to the success of the Department. Please stay in touch and let us know your news. If you happen to be in the area, please let us know — we would be very pleased to have the chance to catch up with you.

– Farshid Hajir
PREDICTING DENGUE FEVER: BATTLES OF THE SIR MODELS
by Shaina Rogstad

Uncertainty Quantification: The Story Begins

This year’s Applied Math Masters project utilized the emerging field of uncertainty quantification to focus on a topic of concern to human health: developing a Susceptible-Infected-Removed model (SIR) for forecasting the spread of dengue hemorrhagic fever (DHF). The project was a collaborative effort by students Simon Burhoe, Cassie DePietro, Peng Du, Rachel Gordon, Domonic Mei, Shaina Rogstad, Thananya Saksuriyongse, Ankita Shankhdhar, and Cortney Tilley under the guidance of Professor Markos Katsoulakis.

Uncertainty quantification (UQ) is an exciting synthesis of mathematics, statistics, and computation aimed at assessing the impact of variability and missing data on model simulations. Understanding the sources of uncertainty inherent to a line of inquiry is key to correctly interpreting the predictions of any system model. The techniques of UQ have wide ranging applications to many areas of study such as climate models, weather models, and biological models. With such broad reaching applications it was difficult at first to narrow down what the focus of the Applied Math Masters project would be. During the first semester of the project the students learned about these applications, while outside of the classroom many participated in the Hack Ebola conference. Biological models quickly became the focus of the project as the students learned more about the spread of infectious diseases through the conference, the news, and a paper that Professor Katsoulakis introduced them to. The paper involved an SIR model of dengue fever in Senegal and instantly resonated with many of the group members. The project topic had been found.

Dengue hemorrhagic fever (DHF) is a mosquito-borne viral infection which has been on the rise in recent decades. There are as many as 36 million symptomatic cases worldwide each year. It is found in tropical regions where ample rainfall creates habitats for mosquitos. While it has no vaccine or cure, dengue fever is treatable. However DHF, the more severe strain, is still a major cause of death in susceptible populations, especially among children. The disease has been spreading in recent decades as climate patterns shift and has led to a number of severe outbreaks. In addition to the human toll, the spread of DHF has many economic implications. Reduced tourism as a result of DHF health concerns was responsible for a net revenue loss of $363 million in Thailand alone during one recent outbreak. Due to the continued spread of this and other infectious diseases the development of accurate SIR models is an important topic in applied mathematics.

FIG. 1: In addition to the toll on human health and medical systems, there is a large economic toll taken by the spread of DHF.
Developing a Plan of Attack

To begin working on the project the nine students were divided into three teams: data, pre-regression, and regression. The teams form an interconnected network that enables an iterative process to obtain the most meaningful results. The process works by the data informing the models which then yield predictions. Utilizing these predictions with new data processing in turn produces a better model and more accurate predictions.

The data team was in charge of obtaining and reducing the data. This is where the challenges began. A promising initial approach was to use the World Health Organization’s DengueNet, but it proved to be too difficult to obtain any usable data from there. The data team next contacted the authors of several papers on the subject of SIR models for dengue. The researchers were very busy, and there were many delays in correspondence, but ultimately the data team obtained a fruitful piece of information, the name Professor Nick Reich. Professor Reich is a professor of Biostatistics and Epidemiology here at UMass Amherst. The data team met with him in Arnold House and was finally able to obtain quality data. He had been collecting it from the Thailand Ministry of Health for many years for his own research, and while it was not publicly available, he graciously allowed the team to use it. The data contained total infection counts and counts by region in yearly and monthly formats spanning the time period 1980–2005.

The monthly data was more sparse than what the data team was hoping to model. As a result, the students learned about various forms of data imputation and developed a script to generate biweekly data by performing stochastic regression imputation. In addition they implemented time series analysis and calculated the power spectral density to reduce the noise and assess the primary frequencies. The dominant frequency seen in the PSD is one year, which demonstrates the annual cycle of dengue infections in Thailand. Prominent
frequencies for cycles less than one year were also observed, leading to speculation that seasonal precipitation and climatological variations could be affecting the breeding cycle of mosquitos and the overall number of DHF cases.

The Battle of the Models

On the model-building side of the project, the pre-regression team was responsible for parameter selection. Their goal was to reduce the number of parameters in the model in order to achieve easier data-fitting for the model being built by the regression team. The pre-regression team applied sensitivity analysis methods from UQ to do the reduction by ranking each parameter. Before ultimately building their own Matlab code, the students attempted several approaches for the sensitivity analysis such as Copasi, which failed due to the lack of chemical reactions in our topic, and Simbiology, a Matlab toolbox that ended up being too costly to obtain. The code built by the students creates sensitivity functions with respect to each parameter and solves the resulting systems of ordinary differential equations. Then the pre-regression team found the relative sensitivity of each parameter and was able to rank them. The ranking was passed along to the regression team to optimize those parameters within their model.

The goal of the regression team was to compare the data collected by the data team and find a good approximating model. The model from Senegal was used as an outline since a model of the type they were interested in was not currently published for Thailand. To strengthen the model from Senegal, the regression team built a program that performed a least-squares regression on the model and solved the ordinary differential equation from Senegal with the parameters obtained at each step of the least-squares regression. However, it proved to be too computationally expensive to perform this algorithm on a 9-dimensional parameter space. Relying on the results of sensitivity analysis, the regression team obtained 4 parameters deemed the most important. With renewed vigor, this team was able to use its algorithm to build a “best-fit” model over the entire data set. Unfortunately, the students were unable to consistently predict the severity of outbreaks using this method, and so they performed the regression in yearly intervals, obtaining a set of parameters dependent on time. While it could be argued that this was an over-fitting of data, the relationship between the parameters and time could give an invaluable understanding of how the biological parameters influence the outbreaks of DHF. Further investigations are needed to clarify this influence.

The regression team ultimately built three models that were able to capture the periodicity of outbreaks and to predict the duration by using the data from previous years. However, the models did not predict the intensity of outbreaks in the subsequent year. Future work would focus on predicting outbreak intensity and expanding the prediction to longer time periods by utilizing enhanced modeling approaches, a more robust data set, and taking into consideration variability due to climate factors.

The students would like to thank Professor Katsoulakis for his support and encouragement and Professor Reich for supplying the data set which made this work possible.
Figure 5. Solution of SIR model. This figure compares 3 different solutions: the baseline value parameter (---), the solution which perturbs the birth rate by 10% (--), and the solution which perturbs birth rate by -10% (-). This demonstrates that the birth rate has a large impact on the solution and has the potential to destroy the limit cycle. This picture confirms our sensitivity analysis that birth rate is the most important parameter.

Figure 6. Sensitivity Function for Infected (Human) Primate. This figure shows how each parameter affects the number of humans infected with DHF throughout time. The higher the peaks the greater the impact that parameter has on the model. We take six most important parameters into consideration and use the obtained ranking to improve performance in the regression model.
Recent advances in big data and data science have introduced new challenges for statisticians, due to the exponential growth of information being generated. A broad range of application areas are affected by big data, including genomics, healthcare, finance, sustainability, climatology, astrophysics, and energy, among many others. The term big data refers to data sets that are either too large or too complex to be analyzed by traditional statistical methods. One primary difficulty in analyzing these large data sets is the limitation on file sizes that can be read into computer memory; another difficulty is that files are often stored and processed on more than one machine due to their massive sizes. In our work, we develop statistical methods and computational tools that overcome these restrictions; this research is in collaboration with Visiting Assistant Professor Alexey Miroshnikov in our department and Evgeny Savel’ev of the Department of Mathematics at Virginia Tech.

Our research in big data focuses on the development of Bayesian statistical models and analysis methods. Bayesian statistics is a statistical paradigm where unknown model parameters are estimated based on the combination of observed data and prior beliefs about the model parameters; the results are called posterior distributions. Bayesian models can be implemented through Markov chain Monte Carlo sampling, which is a technique for generating samples from a distribution. A typical analysis will produce 10,000 posterior samples for each model parameter; these samples are summarized through means, quantiles, and histograms. For data sets that are too large to be analyzed in their entirety, our work focuses on developing Bayesian parallel computing methods for subsets of data. The subset analyses are combined to estimate results based on the full data sets.

Our research goals are to improve on several existing methods that were recently developed by statisticians and computer scientists primarily at Google and Carnegie Mellon University. These current methods use either weighted averages of the subset samples or smoothing techniques to combine subset samples to estimate full data samples. For weighted averaging, the subset posterior samples are weighted by their variabilities; this method is named Consensus Monte Carlo. The kernel smoothing approach is a statistical method that represents a set of points as a smooth line or surface.

For Bayesian subset posterior samples, the kernel smoothing procedure combines the subset samples to form an estimate of the full data surface.

We recently developed a software package to carry out these current Bayesian methods for big data, using the freely available statistical programming language R. Our package is named `parallelMCMCcombine` and is freely available from the Comprehensive R Archive Network (CRAN) at [http://cran.r-project.org/](http://cran.r-project.org/). To demonstrate the use of this package, we analyzed a large data set of all commercial flights within the United States for the 3-month period from November 2013 through January 2014. Our goal was to estimate the average flight delay in minutes for 329,905 flights with arrival delays greater than fifteen minutes. We estimated the two model parameters \( \alpha \) and \( \beta \) of the Gamma distribution, which are the shape and scale parameters, respectively. For this data set, we randomly split the data into five equal subsets, and performed parallel Bayesian analysis on the subsets. A full data analysis was also carried out for comparison purposes.

Results are displayed for the \( \alpha \) parameter in Figure 1. To combine the five subset posterior samples, we show results for the Consensus Monte Carlo algorithm versus the full data results in Figure 2. We compare the two distributions using an estimated relative distance metric; this value is zero when two distributions are exactly the same. For the \( \alpha \) parameter results in Figure 2, the estimated relative distance...
is 0.02 between these two distributions. This indicates that the Consensus Monte Carlo algorithm produces an estimated full-data posterior distribution that is close to the true full data posterior distribution. The estimated relative distance metric is used to compare all methods and determine the best-performing method for different models and data sets. Further data sets, models, and results can be found in our recent *PLOS ONE* paper (Miroshnikov and Conlon, 2014, “parallelMCMCcombine: An R Package for Bayesian Methods for Big Data and Analytics”, *PLOS ONE* 9(9): e108425.)

In additional research in big data, we are developing Bayesian statistical models that depend only on summary statistics of data, rather than complete data sets. The goal of these models is to speed analyses and to alleviate computer memory limits for large data sets. Since summary statistics can be calculated on portions of data sets and then combined, our methods allow for analysis of multiple sources of data and for data that is updated over time, such as online streaming data. We have implemented these methods for Bayesian linear regression models for big data, in the R package named BayesSummaryStatLM, which is also available on CRAN. This package reads in big data sets in analyzable chunks while simultaneously calculating summary statistics; it then performs Bayesian linear regression analysis based on the combined summary statistics. We are currently working on further Bayesian statistical models for big data that use only summary statistics as input.

For activity in big data and data science at UMass Amherst and across the UMass system, there are many opportunities for statisticians and mathematicians to become involved. Potential collaborators include researchers in computer science, biostatistics, public health, management, computational social science, climatology, and education, among many others. Students can become involved in recurring Five College DataFest events, which are 48-hour big data analysis competitions with teams of five students http://www.science.smith.edu/departments/math/datafest/. There is also a new graduate student group GRiD (Graduate Students interested in Data), which is currently co-chaired by Konstantinos Gourgoulias, an applied mathematics graduate student in our department, and Emily Ramos of Biostatistics; information is available at http://umassamherst-grid.github.io/. In other activity, the statistics faculty in our department is developing new computational statistics courses to begin in 2015–2016, and the School of Computer Science at UMass Amherst opened a new data science center in spring 2015.

**FOUR SIMONS FELLOWS IN OUR DEPARTMENT**

Professor Hongkun Zhang was named a 2015 Simons Fellow in Mathematics (see back cover article). This award, given to approximately 40 mathematicians in the United States and Canada each year, enables research leaves by extending a one-semester sabbatical to a full year (at full pay), and thus provides researchers with time away from classroom teaching and academic administration. The award is given by the Simons Foundation, a private foundation based in New York City and incorporated in 1994 by Jim and Marilyn Simons. The Simons Foundation's mission is to advance the frontiers of research in mathematics and the basic sciences, including focused initiatives on the origins of life and the causes and diagnosis of autism. This is the fourth time in four years that a member of the Department has won a Simons Fellowship, an achievement shared by only two other universities, University of California, Berkeley and University of Michigan. The previous three Simons Fellows in the Department were Ivan Mirkovic in 2014, Andrea Nahmod in 2013, and Paul Gunnells in 2012.
**HOW DO WE RECOGNIZE A PERFECT SQUARE?**

by Siman Wong

Number theory is the study of integer solutions of polynomial equations. Many of its problems, such as Fermat’s Last Theorem, have long and distinguished histories. At the same time, the subject is also at the forefront of contemporary research and has important applications to telecommunications and other fields. In this article I illustrate the kind of problems studied by number theorists, the tools involved in solving them, and the interconnections among these topics and applications by way of the following question: How do we recognize a square? In other words, given a positive integer $n$, can we find an integer $m$ such that $n = m^2$?

Upon hearing this question, one’s first response is probably “just use a calculator to calculate $\sqrt{n}$ and see if the answer is an integer.” However, the following two implicit assumptions underlie this response:

(i) There is an efficient way to compute the square root of a number.

(ii) If, for example, the first 10 digits in the decimal expansion of $\sqrt{n}$ are 0, then all the digits in the decimal expansion of $\sqrt{n}$ are 0, and thus in fact $\sqrt{n}$ is an integer.

I will come back to assumption (i) at the end of this article; for now let us focus on assumption (ii).

The first observation is that not every integer is a square, so our original question is not vacuous. More generally, not every integer is the square of a rational number, i.e., a fraction $a/b$ where $a$ and $b$ are both integers. As a concrete example, we prove that $n = 2$ is not the square of a rational number. Assume to the contrary that 2 is the square of a rational number $a/b$; we can assume that at least one of the integers $a$ or $b$ is odd. Then $2 = (a/b)^2 = a^2/b^2$, and so $2b^2 = a^2$. Since $2b^2$ is an even integer, it follows that $a^2$ is even and hence that $a$ is an even integer having the form $a = 2A$ for some integer $A$. But then $2b^2 = (2A)^2 = 4A^2$, from which it follows that $b$ is even. Since this contradicts the assumption that at least one of the integers $a$ or $b$ is odd, we have completed the proof that 2 is not the square of a rational number.

We restate this conclusion quantitatively as follows: the gap between $\sqrt{2}$ and any rational number $a/b$ is not zero.

We can sharpen this conclusion substantially if we restrict ourselves to a certain infinite subset of rational numbers. The proof of the following theorem, which was discovered by Dirichlet, is not difficult.

**Theorem 1** (Dirichlet Gap Principle). There are infinitely many rational numbers $a/b$, where $a$ and $b$ have no common factors, such that the gap between $\sqrt{2}$ and $a/b$ is at most $\pm 1/b^2$.

The rational numbers $a/b$ furnished by the gap principle in Theorem 1 necessarily have denominators that are arbitrarily large. We leave this assertion to the reader as an exercise. Using the gap principle in Theorem 1, we can now address assumption (ii) in the second paragraph. Let $a/b$ be any one of the infinitely many rational numbers furnished by the gap principle. Then the gap between $b\sqrt{2}$ and $a$ is at most $\pm 1/b$. Letting $b$ become arbitrarily large, we have now produced infinitely many irrational numbers of the form $b\sqrt{2}$ whose distance from the closest integer is as small as we want. However, $b\sqrt{2}$ is an irrational number and therefore not an integer. This argument reveals that assumption (ii) is incorrect. We draw the following lesson from this discussion.

**Lesson #1**

a) Be careful about relying on calculators or computer software to determine if a real number is in fact an integer. We can rectify this issue by using suitable error estimates, a standard practice in scientific computing. In Lesson #2(c) we will revisit this in the form of effective results.

b) To study questions about integers it is often helpful to bring in auxiliary quantities such as rational numbers and more general quantities such as elements of global fields.

c) Quantitative results such as the gap principle in Theorem 1 concern the real line, which is a continuum.

Just because calculator values alone are not sufficient to determine whether a given integer is a square does not mean we should abandon numerical investigations. Quite the contrary. In fact, number theorists have a long and distinguished history of using experimental data to formulate conjectures and to gain insights in their research. To illustrate this point let us compute the squares of a few integers and see what pattern might emerge.

<table>
<thead>
<tr>
<th>$m$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m^2$</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
<td>36</td>
<td>49</td>
<td>64</td>
<td>81</td>
<td>100</td>
<td>121</td>
<td>144</td>
<td>169</td>
<td>196</td>
<td>225</td>
<td>256</td>
<td>289</td>
</tr>
</tbody>
</table>

Table 1. Squares of integers
Focusing on the last digit of \( m^2 \) reveals something interesting. There are 10 possibilities for this last digit; i.e., the 10 digits between 0 and 9. However, of these 10 possibilities the only digits that appear are 1, 4, 9, 6, 5, 0. Now the last digit of an integer is none other than the remainder of that integer when divided by 10. The only reason we use 10 is because we are accustomed to the decimal notation. What would happen if we use other bases? There are two comments before we proceed. First, because of the Fundamental Theorem of Arithmetic, number theorists like to work with prime numbers, which we recall are numbers that are not divisible by any integer except itself and 1. So let us try prime bases. Second, given an integer \( a \) and a prime number \( p \), to simplify the exposition we introduce the concept of \( a \) modulo \( p \), defined as

\[
 a \pmod{p} := \text{the remainder when } a \text{ is divided by } p.
\]

With this definition in mind, here are the possible remainders for various odd prime bases:

<table>
<thead>
<tr>
<th>Prime ( p )</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>11</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a \equiv m^2 \pmod{p} )</td>
<td>0, 1</td>
<td>0, 1, 4</td>
<td>0, 1, 2, 4</td>
<td>0, 1, 3, 4, 5, 9</td>
<td>0, 1, 3, 4, 9, 10, 12</td>
</tr>
</tbody>
</table>

Table 2. Remainders for various prime bases

Again, as we noted after Table 1, we get only about half of the possible remainders — in fact, exactly half of the non-zero remainders modulo \( p \). This leads us to define the following symbol: for any integer \( n \) satisfying \( n \equiv m^2 \pmod{p} \), define

\[
 (n/p) := 1 \text{ if there exists an integer } m \text{ such that } n = m^2 \pmod{p},
\]

\[
 (n/p) := -1 \text{ if there does not exist an integer } m \text{ such that } n = m^2 \pmod{p}.
\]

For example, when \( p = 5 \), \( n/p = 1 \) for \( n = 1, 4 \) and \( n/p = -1 \) for \( n = 2, 3 \).

If an integer \( n \) not divisible by \( p \) is an actual perfect square, then certainly \( (n/p) = 1 \). Thus, given an integer \( n \), if we can find a prime \( p \) such that \( (n/p) = -1 \), then \( n \) is definitely not the square of an integer. But can every non-square number be detected in this way? After all, because there are infinitely many prime numbers, we cannot simply check all possible symbols \( (n/p) \). To overcome this obstacle, we bring in a fundamental result in classical number theory due to Gauss.

**Theorem 2** (Gauss’s Law of Quadratic Reciprocity). Let \( p \) and \( q \) be distinct odd prime numbers. Then the product 

\[
(p/q) (q/p) = (-1)^{(p-1)(q-1)/4}.
\]

To see how this law helps us determine if an integer \( n \) is a square, let us examine the case \( n = 5 \). By quadratic reciprocity, 

\[
(5/p) = (p/5)\text{ for any odd prime } p \neq 5.
\]

This reduces the problem of finding a prime \( p \) for which \( (5/p) = -1 \) to the problem of finding a prime \( p \) for which \( (p/5) = -1 \). But we saw in Table 2 that for any integer \( a \), prime or otherwise, \((a/5) = -1\) precisely when \( a \equiv 2 \pmod{5} \) equals 2 or 3. So to show that 5 is not a square, we are led to find a prime number \( p \) whose remainder when divided by 5 equals 2 or 3. Such primes are furnished by another fundamental result in classical number theory, again due to Dirichlet.

**Theorem 3** (Dirichlet’s Theorem on Primes in Arithmetic Progression). Let \( 1 \leq a < n \) be an integer having no common factor with \( n \). Then there are infinitely many prime numbers \( p \) satisfying \( a = p \pmod{n} \); i.e., \( p \) has remainder \( a \) when divided by \( n \).

If we combine the two fundamental results stated in Theorem 2 with \( q = 5 \) and in Theorem 3 with \( n = 5 \), then we conclude that 5 is indeed not a square — certainly, a convoluted argument having an obvious conclusion! But with minimal modification it readily turns into a deterministic algorithm for determining if a given integer \( n \) is indeed a square based on the symbols \( (n/p) \). For this algorithm to be practical we need to address one more issue. Given a non-square integer \( n \), how many prime numbers \( p \) do we have to search in order to find a prime \( p \) satisfying \( (n/p) = -1 \)? An answer is given in the next theorem. The second assertion involves advanced material including the General Riemann Hypothesis, which is explained in footnote 1.

**Theorem 4** (Effective Dirichlet’s Theorem on Primes in Arithmetic Progression). There exist positive constants \( A \) and \( B \) such that for any non-square integer \( n \), there exists a prime \( p < A|n|^B \) satisfying \( (n/p) = -1 \). Furthermore, if we assume the Generalized Riemann Hypothesis for Dirichlet L-functions, then exists an explicit positive constant \( C \) such that \( p < C|\log |n||^2 \).

From Theorems 3 and 4 we gain additional insight, which is summarized as follows.

**Lesson #2.**

a) When applicable, reciprocity laws convert an infinite problem involving all primes to a finite problem involving finitely many primes.
b) The existence of primes required by reciprocity laws is often proved using equidistribution results such as Dirichlet's theorem on primes in arithmetic progression.

c) Effective density results such as Theorem 4 are crucial in applying these techniques to solve concrete problems.

d) Based on our current knowledge, optimal effective estimates often rely on some form of the Riemann Hypothesis\(^2\).

As is shown by the discussion of recognizing a perfect square, reciprocity laws and equidistribution results go hand-in-hand in many problems in number theory. The systematic study of these two topics is codified in the theory of Galois representations and the theory of L-functions.

We end this article by returning to assumption (i) in the second paragraph, which concerns the efficient and quick computation of square roots of integers. Computing square roots of a real number is a classical problem dating back to the Babylonians. It is routinely taught in calculus as an application of Newton’s method. However, as we hinted in Lesson #1(b), it is often useful and instructive to study the same arithmetic problem using other types of real numbers. Here is a concrete example. If an integer \( n \) is the remainder modulo \( D \) of a square, what are the square roots of \( n \) modulo \( D \)? If \( D \) is a prime number, then there is a indeed an efficient algorithm for finding all square roots modulo \( D \), based on the deep theory of elliptic curves over finite fields. However, if \( D \) is the product of two distinct prime numbers, there is no known, efficient algorithm to find all the square-roots modulo \( D \). Our inability to do so is one of the key theoretical underpinnings of the RSA public key cryptosystem. Similarly, while every scientific calculator has a logarithm button, there is no button for computing logarithm modulo \( D \), even if \( D \) is a prime. Here the lack of such an algorithm is one of the key theoretical underpinnings of the various discrete-log-based cryptosystems.

Lesson #3

a) Number theory has important applications to computer science and telecommunications, and questions from these fields in turn give rise to new and exciting problems for number theorists.

b) Computational experiments are crucial for formulating conjectures and developing insights in number theory. To carry out these experiments we need to turn theorems into algorithms; this leads to new theoretical developments, and the end results are often applicable to other fields.

My goal in this article was to give readers an idea about the kinds of problems studied by number theorists, the tools involved in solving these problems, and the applications and connections with other branches of sciences and engineering. I hope that this discussion inspires you to learn more about the fascinating and profound field of mathematics known as number theory.

\(^1\)The Riemann Hypothesis (RH) concerns the roots of a power series known as the Riemann zeta function, built using prime numbers. The Generalized Riemann Hypothesis (GRH) concerns the roots of a related power series whose coefficients are weighted by symbols such as \((n/p)\). RH and GRH have been extensively studied, both theoretically and computationally, for over a century. RH is one of the seven Millennium Prize Problems selected by the Clay Mathematics Institute in 2000. The solution of each Millennium Problem comes with a prize of one million US dollars and would lead to significant advances in mathematics and beyond.

\(^2\)Being able to appeal to RH is often a non-trivial accomplishment; many problems cannot be resolved even under RH. Here is a concrete example. Take an \( N \times N \) grid, and put the integers 1, 2, \ldots, \( N^2 \) successively in each slot, starting from the upper left hand corner and moving down one row at a time. Is there always a prime in each row if \( N > 1 \)? This unsolved problem is barely beyond the reach of RH, and an affirmative answer has important implications in algorithmic number theory.

CLIMATE CHANGE, STATISTICAL SIGNIFICANCE, AND SCIENCE: WHY THE NEW YORK TIMES OPED PAGE NEEDS A STATISTICIAN

In January 2015 The New York Times published an opinion piece, “Playing Dumb on Climate Change,” by Naomi Oreskes, which argues that in the case of climate change, scientists are too conservative in their scientific standards. Scientists adhere to standards that call for 95 percent confidence levels, or \( p \)-values of less than 5%. In Oreskes’ opinion, scientists ought to use a standard less stringent than 95 percent because there is a plausible causal mechanism for climate change and because the risk from inaction is so great. Unfortunately, to make her argument, she confuses several different aspects of confidence, evidence, belief, and decision-making. Michael Lavine, professor of statistics in
our department, put a commentary about Oreskes’ piece on http://www.stats.org, a website devoted to good statistical practice in journalism. Below are excerpts from Lavine’s comments. Quotes from Oreskes are in italics, and responses from Lavine in plain text. See http://www.stats.org/climate-change-statistical-significance-and-science/ for the full commentary.

Oreskes writes that “scientists apply a 95 percent confidence limit, meaning that they will accept a causal claim only if they can show that the odds of the relationship’s occurring by chance are no more than one in 20.” The interpretation of “relationships occurring by chance” requires care. Suppose data show that the last decade was warmer than usual. Then the 95% confidence limit means Prob{decade this warm or warmer} / Prob{decade cooler than this} ≤ 1/19, where the probabilities are calculated under the assumption of no climate change. It does not mean that P1/P2 ≤ 1/19, where P1 is the probability that this warm decade was caused by chance and P2 is the probability that this warm decade was caused by climate change.

Also, Oreskes is wrong to say that scientists treat the 95 percent confidence limit as a “causal claim.” The confidence level and the confidence interval speak to the probability that we would see temperatures this warm or warmer if they were simply random fluctuations. No causes can be inferred without further information. It seems the BBC made a similar mistake a few years ago, when it attempted to describe research results declaring global warming statistically significant, stating that “scientists use a minimum threshold of 95% to assess whether a trend is likely to be due to an underlying cause, rather than emerging by chance.” The BBC article is available at http://www.bbc.co.uk/news/science-environment-13719510.

Oreskes continues, “there have been enormous arguments among statisticians about what a 95 percent confidence level really means.” That’s not right; almost every statistician knows what a 95 percent confidence level means. But many working scientists, and Oreskes, a historian, get it wrong.

Later, Oreskes says, “But the 95 percent level ... is a convention, a value judgment. The value it reflects is one that says that the worst mistake a scientist can make is to think an effect is real when it is not. This is the familiar ‘Type 1 error.’ You can think of it as being gullible, fooling yourself, or having undue faith in your own ideas. To avoid it, scientists place the burden of proof on the person making an affirmative claim. But this means that science is prone to ‘Type 2 errors’: being too conservative and missing causes and effects that are really there.” A Type I error would be labeling “significant” a decade that is warm merely because of random fluctuation. A Type II error would be failing to label a decade that is warm because of climate change. As Oreskes notes, we can decrease the number of Type I errors by using a stricter standard for labeling, but only at the expense of increasing the number of Type II errors. Or we can decrease the number of Type II errors by adopting a looser standard, but that would increase the Type I errors.

Oreskes continues, “Is a Type 1 error worse than a Type 2? It depends on your point of view, and on the risks inherent in getting the answer wrong.” But there are no risks associated with either Type I or Type II errors. Risks arise only when we take actions. Type I and Type II errors do not prescribe actions; they describe whether data is consistent with chance mechanisms. Type I errors occur when data generated by chance appear to be inconsistent with chance. Type II errors occur when data not generated by chance — at least not by a null or uninteresting chance mechanism — appear to be consistent with chance.

“What if we have evidence to support a cause-and-effect relationship? ... Then it might be reasonable to accept a lower statistical threshold...” A lower threshold for what: confidence, beliefs, or action? Oreskes talks about 95 percent confidence, but she also seems to be calling for us to accept the reality of climate change and to do something about it. To statisticians, confidence, beliefs, and thresholds for actions are different things. Confidence is about the probability that chance mechanisms can produce data similar to, or even more extreme, than the data we’ve seen. Beliefs have to do with assessing which is really responsible for the warm decade: chance or climate change. Action thresholds depend on our beliefs but also on costs, risks, and benefits. By not distinguishing between confidence, beliefs, and actions in her call for a lower threshold, Oreskes helps perpetuate the confusion surrounding these concepts.

After urging scientists to adopt a threshold less stringent than 95 percent in the case of climate change, the piece continues, “WHY don’t scientists pick the standard that is appropriate to the case at hand, instead of adhering to an absolutist one? The answer can be found in a surprising place: the history of science in relation to religion. The 95 percent confidence limit reflects a long tradition in the history of science that valorizes
skepticism as an antidote to religious faith. Even as scientists consciously rejected religion as a basis of natural knowledge, they held on to certain cultural presumptions about what kind of person had access to reliable knowledge. One of these presumptions involved the value of ascetic practices. Nowadays scientists do not live monastic lives, but they do practice a form of self-denial, denying themselves the right to believe anything that has not passed very high intellectual hurdles.”

Yes, most scientists are skeptics. We do not accept claims lightly, we expect proof, and we try to understand our subject before we speak publicly and admonish others.

Thank goodness.

— Michael Lavine

SOLUTIONS TO LAST YEAR’S CHALLENGE PROBLEMS

Your proliferating Problem Master (PM) Rob Kusner is about to take a sabbatical, so he’s happily passing the baton to colleague Franz Pedit, who helped create the next round of Challenge Problems. This year’s problem solvers included Doug Bosworth (MA ’88), John Cade (PhD ’78), Dean Jordan (BA ’66), Mark Leeper (BS ’72), Peter Miner (BS ’80) and Cort Shurtleff (BS ’79). Dean expressed “thanks to you and your colleagues for providing these entertaining problems,” and Cort praised the “wonderful teachers” he had at UMass Amherst, while urging our department to give ample “attention to undergrads” with “appropriate activities” – perhaps like these problems?

Recall that we had a 7-course menu of problems last year, so your PM – being rather full now – offers solution “tidbits” when he feels the reader can fill in the details:

Problem 1. An Amherst farmer divides a square field ABCD by erecting a stone wall along the diagonal AC to form two triangular fields ABC and CDA. The farmer then decides to subdivide the triangular field ABC into two smaller fields of equal area using the shortest straight fence possible. Find – with proof – the length and location of this fence.

One might guess the shortest fence ends at the corner B and at the midpoint of AC (perpendicularly), but one can decrease its length – while preserving the area – by moving one end from B toward A (or C) and the other end toward C (or A). As long as the enclosed angles at the ends differ, the length can be decreased; so one of the small fields enclosed by the shortest fence must form an isosceles triangle, from which the length can be solved; by scaling, it must be a constant times the square-root of the area, and this constant works out to be $\sqrt{\sqrt{2}-1}$.

Problem 2. Let $f(x)$ be a continuous real-valued function such that $f(2014x) = f(x)$ for every real number $x$. Is it true that $f(x)$ is a constant function?

For every integer $n$ and real number $x$, the relation implies $f(x) = f(x/2014^n)$. So as long as $f(x)$ is continuous at $x=0$, we can take the limit as $n \to \infty$ and conclude $f(x) = f(0)$ for all $x$. (If we drop the continuity assumption at 0, there are plenty of nonconstant examples: $f(x) = \sin \left( \frac{2\pi \ln x}{\ln(2014)} \right)$ is one that Mark suggests, inspired by a circular slide-rule – anybody remember this simple mechanical device for realizing multiplication? Your outgoing PM has long-dreamt of a “non-commutative slide-rule” to perform matrix multiplication – your incoming PM may comment next year whether that’s possible – stay tuned!)

Problem 3. In the equation

$$[3(230 + t)]^2 = 492,004$$

find the integer $t$ and the digit $a$.

Expanding the square, we see the last digit of $(3t)^2$ is divisible by 4, suggesting $3t=12$, or $t=4$, so $a=8$. (Dean and Mark each note that trial and error is a pretty quick way to the solution too! Peter points out $t=-464$ also works!)

Problem 4. The graph of $y = x \sin \frac{\pi}{x}$ for $0 < x \leq 1$ defines a bounded (but very wiggly) curve in the $(x,y)$-plane. Is the length of this curve finite or infinite?

Doug considers the path that zig-zags back and forth between the points where wiggly curve meets the envelope $\{y=\pm x\}$; these are at $x = \frac{2}{2n+1}$, where $n$ is a positive integer,
with the sign alternating as $n$ is even or odd. Comparing with the harmonic series $\sum_{n=1}^{\infty} 1/n$ shows the length of the zig-zag path, and of the (even longer) wiggly curve, is infinite.

**Problem 5.** Any $2\times2$ matrix $P = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ is called Pythagorean if $a$ and $b$ are integers such that $a^2+b^2 = c^2$ for some integer $c$. Suppose $Q = \begin{pmatrix} 235 & -411 \\ 411 & 235 \end{pmatrix}$.

Show that $P = Q^{2014}$ is Pythagorean.

Since checking things is just matrix arithmetic, we simply record the main points. Observe (as several readers did) that the product of Pythagorean matrices is again Pythagorean, so we would be done if the matrix $Q$ were Pythagorean – but it is not. What saves the day? The square of any matrix of this form is Pythagorean. And thus $P=Q^{2014}=(Q^2)^{1007}$ is Pythagorean too.

**Problem 6.** An infinite chessboard has squares indexed by integer vectors in the plane. This problem has three parts:

a) A Knight moves by any one of these eight vectors:
   \[ \{ \pm \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \pm \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \pm \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \pm \begin{pmatrix} -1 \\ 1 \end{pmatrix} \} \]

Show that a Knight can visit any square on this chessboard (that is, any integer vector in the plane is an integer linear combination of these eight vectors).

b) An “impaired” Knight – or Knite for short – can only make the first four moves: \[ \{ \pm \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \pm \begin{pmatrix} 1 \\ 2 \end{pmatrix} \} \]

What fraction of the squares can a Knite visit?

c) Suppose a new chess piece – the Mathprof – makes these six (rather weird) moves: \[ \{ \pm \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \pm \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \pm \begin{pmatrix} 5 \\ 5 \end{pmatrix} \} \]

What fraction of the squares can a Mathprof visit?

Cort points out that part a) is standard chess lore; but since we are using vectors, observe (as both Dean and Mark did) that using combinations of 3 Knight moves we can form the basic horizontal vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and the basic vertical vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and thus move anywhere on the (infinite) board. The same idea works for part c) using combinations of 3 Mathprof moves with integer coefficients (no longer just ±1; readers may explore the many possibilities). Part b) is trickier, since the Knite cannot move everywhere. (One thoughtful reader objects that the notion of “fraction” is ill-defined in this infinite setting; however, since things are periodic, it really reduces to a finite problem and the fraction makes sense.) Both Mark and Dean correctly identify the fraction to be 1/3; rather than giving away their solutions, your PM notes it is no accident that 3 is the determinant of the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, and also the area of the fundamental parallelogram spanned by $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

**Problem 7.** Call a function lucky if it can be expressed as a sum of a polynomial function $p(t)$ and a continuous periodic function $q(t)$ with period $2\pi$ (this means that $q(t+2\pi) = q(t)$ for every $t$). Show that any antiderivative of a lucky function is lucky.

Here is the key observation: an antiderivative of a continuous periodic function is continuous, but it would be periodic if and only if the average of the function (over any interval of length $2\pi$) vanished; in general, any antiderivative grows linearly with slope equal to the average of the function over any interval of length $2\pi$. And that’s lucky!
OUTSTANDING UNDERGRADUATES HONORED AT 2015 AWARDS DINNER

On April 22, 2015, the Department of Mathematics and Statistics celebrated the accomplishments of our top students at our annual Awards Dinner. This evening honors the winners of the Jacob-Cohen-Killam Mathematics Competition and the M. K. Bennett Geometry Award, as well as our REU participants, members of the Putnam Competition team, and other students deserving special appreciation. Together with the families and friends of the awardees, we were joined by alumni John Baillieul ’67, James Francis ’86, and Roy Perdue ’73, and by Professor Emeritus Eleanor Killam.

The evening began with refreshments and dinner. Several problems included on the program provoked plenty of lively discussion, mathematical and otherwise. The awards portion of the event opened as our guests were tucking into their chocolate lava cake, with greetings from Department Head, Professor Farshid Hajir. Professor Richard S. Ellis expressed our thanks to all our donors for their generous support of the department's activities. Professor Paul Hacking, this year's emcee, presented each student attendee with a small dodecahedron, which is a polyhedron with 12 pentagonal faces. The dodecahedron, first described in the final book of Euclid's *Elements*, displays remarkable symmetry and has inspired mathematicians for millennia. For example, the symmetries of the dodecahedron are intimately related to the famous theorem of Evariste Galois that the general polynomial equation of degree 5 cannot be solved by algebraic means.

In recent years the department has been able to significantly expand our REU program, thanks to the generous support of Joan Barksdale ’66. Three professors were on hand to describe last summer's REU projects and to explain the basics of our REU program.

Professor Mike Sullivan presented the REU in Pure Mathematics and Financial Mathematics, recognizing Zachary Fox, Aerin Thomson, and Shuang Xu.

The REU in Applied Mathematics was presented by Professor Matthew Dobson, who recognized Gabriel Andrade, Scott Destromp, Ian Fox, John Lee, and Hannah Puisto. Finally, Professor Paul Hacking presented the REU in Statistics, recognizing Shai He and Abhijit Pawar.

The M. K. Bennett Geometry Award is presented to the student who exhibits the best performance in Math 461. This award honors the memory of Professor Mary Katherine Bennett, who earned the first Ph.D. in our department in 1966. After teaching at Dartmouth College, she returned to UMass Amherst for the rest of her career where she encouraged interest in geometry and high school teaching among undergraduates. The year-long course that she developed now covers Euclidean, spherical, and hyperbolic geometry, and is taken by all our math majors in the teaching track.

Professor Inanc Baykur presented Aerin Thomson, the winner of the M. K. Bennett Geometry Award, with some non-orientable knitwear: a Klein-bottle hat and Möbius scarf. The Mobius band is a one-sided, or non-orientable, surface obtained from a strip by making a half twist and identifying the ends. The Klein bottle, named for geometer Felix Klein, is a similar surface obtained from a cylinder by identifying the ends with opposite orientations.

This year we also recognized two runners up, Aleksandr Burkatovskiy and David Reed, who were presented with the books *Love and Math* by Edward Frenkel and *The Poincaré Conjecture* by former Mount Holyoke Professor Donal O'Shea, respectively. These books describe recent breakthroughs in modern geometry: the solution of Thurston's geometrization conjecture in 3-dimensional topology by Grigori Perelman, and the Langlands program in geometric representation theory, which includes work of UMass Professor Ivan Mirkovic.

Robert Ambrose, Batkhuyag Batsaikhan, Aaron Dunbrack, Rajesh Jayaram, Michael Mueller, Kai Nakamura, and Aerin Thomson were recognized for competing in the 2015 Putnam Exam. Professor Jenia Tevelev, who has run the successful Putnam preparation seminar for several years, introduced the students. He confided that the seminar reminded him of a movie montage of new recruits in grueling military training, crawling through mud and over barbed wire, etc.

The program for the dinner included the following Putnam problem from 2002. This problem was selected as the most recent Putnam problem which could be safely attempted by attendees without risking indigestion!

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Jacob-Cohen-Killam award winners, sponsors, and organizers.
Kai Nakamura, Aaron Dunbrack, James Francis, John Baillieu, Richard S. Ellis,
Robert Ambrose, Michael Mueller, Franz Pedit, Rob Kusner

Paul Hacking

John Baillieu, Farshid Hajir, Richard S. Ellis

2015 Volume 30
REU Students and advisors: Franz Pedit, Aerin Thomson, Mike Sullivan, Shuan Xu, Gabriel Andrade, Abhijit Pawar, Zak Fox, Ian Fox, Matthew Dobson, and Rob Kusner

and the winners are...

we(e) mathematicians

David Reed, Aleksandr Burkatovskiy, Paul Hacking, Aerin Thomson, Inanc Baykur

Eleanor Killam and Roy Perdue
Five points are marked on a sphere. Can we divide the sphere into two hemispheres so that four of the points lie in the same hemisphere? Points on the equator count as lying in both hemispheres.

The Jacob-Cohen-Killam Mathematics Competition is named in honor of the memories of Professors Henry Jacob and Haskell Cohen, and of the continuing contributions of Professor Emeritus Eleanor Killam. These three faculty members sparked interest in mathematics among undergraduates through annual mathematics contests. Emcee Paul Hacking noted that many mathematicians, including himself, have been hooked on math at an early age by contests like this.

The competition is open to first and second year students. Each year a few dozen contestants attempt to solve ten challenge problems dreamed up by our faculty members. Former contestants often develop deep ties with the department; some choose mathematics as a double major, while others participate in summer REUs or join the Putnam team. This year the competition was again generously sponsored by John Baillieu ’67, Roy Perdue ’73, and James Francis ’86.

Professor Franz Pedit awarded this year’s first prize of $1,600 to Aaron Dunbrack, the second prize of $1,000 to Michael Mueller, the third prize of $600 to Robert Ambrose, and the fourth prize of $200 to Kai Nakamura.

Professor Pedit highlighted the following problem from this year’s JCK competition, which was also included in the program for the dinner:

Imagine two nails 10 cm apart at the same height on a vertical wall, and suppose we have a long piece of string attached to a picture frame in the usual way. Is it possible to wind the string around the two nails so that the picture hangs safely on the wall, but when either one of the nails is removed (but not both) the picture will fall to the ground?

Graduate Student Awards

Graduate program director Professor Tom Braden recognized the winners of the graduate student awards. Alden Gassert, Luke Mohr, and Julie Rana won the Distinguished Thesis Award, while Tobias Wilson received the Distinguished Teaching Award. Professor Braden noted, in particular, Toby’s work teaching Euclidean geometry to middle-school students in Amherst, above and beyond his teaching duties at UMass.

The evening ended with closing remarks by Steve Goodwin, Dean of the College of National Sciences. The Dean spoke warmly of the successful evening and highlighted the success of our REU program, particularly the joint work of Professor Matthew Dobson and his REU students.

RESULTS OF THE 75TH ANNUAL PUTNAM COMPETITION

The Putnam examination is the oldest, most prestigious mathematical competition for undergraduate students in the United States and Canada. A total of 4320 students from 577 colleges and universities participated in the competition in 2014. The winning teams were from MIT, Harvard, RPI, Waterloo, and Carnegie Mellon.

The examination takes six hours separated by a lunch break. The participants are given twelve problems, which rely on the standard undergraduate curriculum, including analysis, number theory, probability, and algebra. However, they require sophisticated investigation and delicate reasoning. The competition in 2014 was unusually hard even by Putnam standards. For example, there was one problem that nobody solved and three other problems that were solved by only five participants. The median Putnam-exam participant didn’t solve any problems.
One of my favorite classes to teach is Math 370. Known officially as “Writing in Mathematics,” this course satisfies the requirement in Junior Year Writing (JYW). Teaching this course gives me an opportunity to meet a group of math majors and to help them explore how writing in mathematics is essential to a well-rounded mathematical education. I also use some of my time with them to help prepare them for the job market. It is exciting to see all the possibilities that students have for their future and the various paths that they can take.

During the past several years I have adopted, as the primary pedagogy, team-based learning into most of my courses. I have seen amazing results and am well versed not only in the benefits of group work, but also in the necessity of students having writing and related skills for the job market.

I was excited to be able to offer the JYW course as a team-based learning course during the spring semester. The course changed in several ways from how it has been traditionally taught. Instead of 100% individual writing, it became 60% individual writing and 40% team writing. The enrollment in the course also doubled to 64 students. In order to accommodate all the students, the class met in one of the new team-based learning rooms in the Integrated Learning Center. As there exist multiple tracks in the math major, it made sense to form groups primarily by track — actuarial, applied math, computing, pure, statistics, and teaching — and secondarily by student-reported strengths.

During the first half of the course we worked on team building and giving constructive feedback. We developed class rubrics for group contributions and learned to use group-rating software called ipeer, which is supported by UMass Amherst. Students learned to document their contributions to a larger project, to give meaningful feedback to their peers, and to take on various roles in a group dynamic, including editor, math checker, skeptic, and team leader. The ultimate focus of the group work was research projects in which students researched various problems based on or related to mathematics. An integral part of the research projects was a poster presentation in LGRT 1634 in which students presented their research in a poster-conference fashion. Students prepared professional, academic posters and answered questions posed by faculty members and graduate students who attended.

The list of projects was extensive and impressive with 14 groups sharing projects from all facets of mathematical research. Projects included “An Optimized Model for Investing,” “Knot Theory and Unknotting the Problem,” “Mathematical Modeling of Hurricanes,” “Narayan Numbers,” “Optimizing Sports Scheduling,” and “The Mathematical Modeling of Sleep.” The students in this year’s class exceeded all expectations and presented truly high-quality work. Their accomplishments were made possible by the team aspect of the course. While the projects would have probably been beyond the capacity of any individual working alone to complete, working in groups allowed the students to perform far better than anyone could have anticipated.

I am hopeful that this tradition of team work will continue in the JYW course. I know that it certainly will be the next time I teach it!

Thirteen students at UMass Amherst took the examination in 2014. Our top participants were Aaron Dunbrack and Rajesh Jayaram, who were listed in the top 500 list sent to all colleges and universities in the United States and Canada. Other UMass participants with distinguished performance were Michael Muller, Andrey Smirnov, Batkhuyag Batsaikhan, and Aerin Thomson.

For further information about the Putnam competition and the problem-solving workshop offered by the Department of Mathematics and Statistics, see http://people.math.umass.edu/~tevelev/putnam/putnam2014.html. Problems, solutions, and winners of recent competitions can be found at http://kskedlaya.org/putnam-archive/.

MATH 370: WRITING IN MATHEMATICS
by Adena Calden
ACTUARIAL SCIENCE: COURSES AND CAREERS

Actuarial science is now among the largest concentrations that the Department of Mathematics and Statistics offers. It is also the fastest growing. The department has over 90 majors in the actuarial concentration, including double-concentrators and double-majors. Our graduates are successful in obtaining positions at top insurance companies, and we are competitive in recruiting high school seniors, who choose UMass Amherst over more established programs.

The fall Actuarial Career Fair, run in conjunction with the Actuarial Club, is an important vehicle for students seeking internships and permanent positions. Professor Anna Liu started the fair with Professor Michael Sullivan, who recently turned over direction of the actuarial program to Professor Eric Sommers (esommers@math.umass.edu). Anna and Mike built the program, established the concentration, and ensured that we are able to offer preparation courses for the first two actuarial exams; their work is greatly appreciated.

Our donors this year included The Hanover Insurance Group, the Actuarial Club of Hartford and Springfield, and alumnus Robert Pollack '54. Donor funds were used to reimburse student exam fees, reimburse exam study material costs for students, cover the career fair expenses, and host events from company representatives. We had a fruitful April visit from our Casualty Actuarial Society liaisons, Chad Wilson and Hayley Shi '10, from Travelers, made possible by Robert Pollack's generous donation. We are extremely grateful for the support of our donors!

This year we instituted an advisory board for the actuarial program. Board members suggest improvements to the program and are another resource for students seeking internships and permanent positions. We asked the five board members, all recent graduates, to tell us a little about themselves and their career paths. The board members are Larry Cantwell '13 (Mass Mutual), Kevin Cavanaugh '14 (Mercer), Lev Kamenetsky '10 (Hanover), Doug Kanner '13 (Aon Hewitt), and Hayley Shi '10 (Travelers).

Why did you choose the actuarial concentration?

Doug: As a sophomore, before I transferred to UMass, I had a teacher who had worked for some years as an actuary, and the conversations I had with him really opened my eyes to the actuarial career. I ultimately chose the actuarial concentration

SPRING 2015 STATISTICS PROJECT SEMINAR

Six graduate students and one undergraduate took the statistics project seminar in the spring semester. Professor John Staudenmayer organized the course and advised three different project groups.

In the first group, Yue Chang and Yifeng Wu worked on an image comparison project with data and questions that came from Professor Dhandapani Venkataraman (“DV”), who is in the Department of Chemistry at UMass Amherst. The project involved comparing scanning and transmission electron microscope images of molecular surfaces with images that came from computer simulations. In addition to learning about methods to turn images into data, the statistics students used non-parametric goodness of fit test methods and simpler exploratory methods to compare the images.

Dongah Kim and Shuang Xu, an undergraduate, formed the second group, and they worked on a factor analysis project suggested by Professor Michael Lavine, who is in our department. After learning the basics of factor analysis and other decomposition methods for multivariate data, the group explored the question of how to construct different datasets that result in the same factor analysis. This work also led to the development of new diagnostic tools to understand the relations between the data and the simplified model produced by the factor analysis. We hope to continue this work over the summer.

Finally, Jun He, Jingyao Hou, and Li Wang worked on a fisheries management project sponsored by Professor Andy Danylchuk, who is in the Department of Environmental Conservation at UMass Amherst. Professor Danylchuk and his students are interested in the behavior of sport fish after they have been caught and released. One way they investigate this is by attaching accelerometers to the fish when they are caught. After the fish swim away, the accelerometers record data about their movement. The accelerometers eventually separate from the fish, float to the surface, and the investigators download the data. The statistics part of the project involved considerable data management and visualization tasks, and the group also learned to use wavelet models to identify different types of swimming behavior. We hope to continue this collaboration as well.
because I was looking for a way to combine my interests in economics, real-world problem solving and risk management with my love of math. The concentration definitely set me up for success, and I am excited to see the actuarial program at UMass continue to develop and expand.

**Kevin:** I decided to choose the actuarial concentration before I came to UMass because my goal was to have the #1 job in the U.S.! Besides the great job outlook and low stress work environment, one of my favorite things about the actuarial career is that you get to determine your own success by working hard and passing your exams.

**Larry:** The actuarial profession offers a great opportunity to have direct control over career progression. Once the exams are completed, it also offers a career with a decent balance between work and life.

**Hayley:** I was fortunate to have figured out early on that my personal motivation lies in solving real-world problems with tangible applications, and it sounded to me like that was what the actuarial career offered. I studied art and architecture for two years at UMass, but eventually gravitated back to the quantitative side. I learned a lot in my art and design classes, and I use what I learned to this day in my presentations, but math was what came naturally to me. You have probably seen the various articles and surveys ranking actuary as one of the most desirable careers in America, citing reasons such as attractive compensation, low-stress work environment, and stable employment. Today, I can say that those are of course all reasons for my being satisfied with my actuarial job. However, what really drives me and allows me to say that I enjoy what I do is getting to work with smart and motivated people every day and to solve interesting problems in a competitive market that impact millions of customers.

**Lev:** I found I had a passion for math in high school and started researching careers that a math major could get me into. My dad actually suggested I take a look at an actuarial career. The first thing I found was that it was rated the “number-one” job several years in a row due primarily to job security, compensation, and low stress levels. It intrigued me to find out that the actuarial profession focuses on solving real-time business problems in an analytical, logical, and scientific way, which sounded like it would give me a challenging, interesting, and secure career. I declared my major in mathematics with a stats concentration soon after, and then to get additional exposure to business, I added a finance major during my sophomore year.

**What was your favorite Math/Stat course?**

**Doug:** Math 441 (Math of Finance). This was a great introduction to the theory and mathematics behind some common derivatives and hedging strategies, as well as a helpful resource when passing both Exam FM/2 and MFE/3F.

**Kevin:** The Exam FM class with Mike Sullivan. It was a very effective prep course for the exam and helped me gain the basic knowledge I needed to pass Exam FM. Another favorite course on campus was Economics 309 (Game Theory) with Peter Skott.

**Larry:** My favorite course was Math 411 (Group Theory). Though my career choice doesn’t reflect it, I have an affinity for pure mathematics.

**Hayley:** Predictably, I remember fondly my Exam FM class, perhaps because I passed the exam and got an A in the course! It didn’t hurt though that the material was presented in an interesting way, and I had great classmates to study with.

**Lev:** A three-way tie between Math 300 (Introduction to Proofs), Math 411 (Group Theory), and Stat 515 (Probability).

Tell us a little about your current job.

**Doug:** I work as a retirement-consulting actuary for Aon Hewitt. As a retirement actuary, I work with the retirement plans (primarily pension plans and retiree medical/life plans) of several corporate clients to help those clients understand the value of those plans, manage the risks associated with them, and balance business strategy with employee lifetime-income adequacy. I’ve also had the opportunity to do some work on the investment side of retirement as well, working
with our investment-consulting group to help clients manage their retirement liabilities and assets in tandem, and in a way that takes as much risk as possible off the table.

**Kevin:** I work as a retirement-consulting actuary for Mercer. My time so far at Mercer has been very rewarding, and I am constantly learning new things and getting involved in new and interesting projects. My favorite part of the job is coming into the office every day and being surrounded by brilliant, determined colleagues who are experts in their respective fields and are very willing to do whatever it takes to help other people.

**Larry:** I currently work in the Retirement Services Pricing division at MassMutual Financial Group. My team and I build, maintain, and enhance the models we use to price retirement plans. We also provide analysis and advice for complex, high priority new business pricing, and help quantify and maintain the profitability of our existing block of business.

**Hayley:** I work in Personal Lines at Travelers, which means that the products that I work on are sold to individuals and families, in contrast to Commercial Lines, which serve companies. The two main products in personal insurance are home and auto. My current role is a non-traditional one. I do the traditional property/casualty work of helping to price insurance policies and reserve for losses from claims for both home and auto. Depending on the current priorities, my work also involves financial planning, helping to test and implement predictive models, post-monitoring of new product launches, helping out with upcoming product builds, and making tools that incorporate actuarial methods for my partners on state teams to help them evaluate the profitability of the states they are responsible for. I also train new analysts and do recruiting for our Actuarial & Analytics Leadership Development Program, of which I am a proud member. I’m a Jack (or perhaps a Jill?) of all actuarial trades!

**Lev:** I work at Hanover, which is a property and casualty company. My job is to manage the pricing activities for homeowners insurance, personal automobile insurance, and other personal lines in 6 of our 17 states.

**What is one piece of advice for current actuarial students?**

**Doug:** Pass the actuarial exams early and often! It seems like college is a tough time to pass exams, since there are classes, extracurricular activities, and other factors to juggle, but it is absolutely easier to pass exams while still in college than it is to pass them while working full-time! In addition, many companies will really consider only candidates for internships and full-time positions who have already passed one or more exams. The best way to get your foot in the door of an actuarial career is to pass the exams.

**Kevin:** Work hard: the more exams you complete before you graduate, the easier your life will be down the road. Play hard: an actuarial student with the ability to pass exams is a must, but being a great communicator is often a more highly valued skill. Have fun: if you’re not having fun doing what you’re doing, then why are you doing it?

**Larry:** Take practice tests under exam conditions: set a timer, remove all study aids, and work through an entire exam from beginning to end. Building up the mental endurance and improving on test-taking skills is nearly as important as having a firm grasp on the material. Also, I have to recommend becoming familiar with the TI-30XS Multiview Calculator — it’s a huge time saver.

**Hayley:** For students who are just beginning their actuarial studies, there are a wide variety of roles within the insurance industry that do not require exams for internships. These include product management, underwriting, sales, and claims. There may also be opportunities available at your local insurance agencies. Experience in any of these areas would be tremendously useful to students, both to confirm whether insurance is the right industry for them, and to begin to accumulate transferrable work experience.

**Lev:** Take your exams early. This will help you understand if an actuarial career is really something you want to pursue and also gives you a leg up during your job-search and internship-search process. Interviewing is a skill, and like any other skill, it takes practice. The
goal is for you to be able to walk into a high-pressure situation and show the interviewer who you really are.

What is one suggestion for strengthening the actuarial program at UMass?

**Doug:** Include all of the Validation by Educational Experience (VEE) courses within the actuarial track. It is very helpful for an aspiring actuary to have passed all of these courses before graduating. In particular, adding a VEE time-series course to the concentration would be extremely helpful.

**Kevin:** Adding an advanced Excel skills course would help a lot of graduates ease the transition into their daily duties at work. It would also be incredibly beneficial if UMass offered a time series course so that students could satisfy all VEE requirements before they graduate.

**Larry:** Split the exam courses into two semesters: one for teaching the concepts and one that focuses on exam preparation. This seems to be the model that most top actuarial programs utilize.

**Hayley:** In addition to exams and work experience, employers look for a well-rounded candidate with skills outside of math courses and exams. Programming knowledge is highly sought after. I like to look for SAS, R, Python, SQL, and increasingly Hive/Pig. Familiarity with these is preferred, but proficiency in any programming language generally lets us believe that the candidate is capable of learning others. Technically strong candidates become highly coveted when they also demonstrate excellent communication skills. Once a student has passed three or four exams, the incremental value of passing another exam in terms of finding a job is probably less than seeking out either classes or club activities where soft skills and group work can be developed and demonstrated. The program at UMass can facilitate reaching these goals by encouraging actuarial students to choose classes such as Stat 597A (statistical computing) and by helping students develop effective communication skills.

**Lev:** I would recommend additional focus on applying statistics knowledge using computers. Robust uses of SAS, R, Excel, and other predictive-modeling software would all give a big leg up to students. A focus on predictive modeling more generally would also be helpful.

Thanks to the actuarial program advisory board for these insights into their profession and for their advice to current students. We continue to look for ways to improve the program and will act on the advice of the board. We are always looking to hear from alumni working in the actuarial field (and related fields) – please drop us a note to let us know how you are doing. We are currently seeking alumni and other friends of the program who are interested in serving on the actuarial advisory board for next year.

Athena Polymeros article continued from cover events. During one of these experiments, by dropping a small ball off a ramp at a specific angle, the software on our computer drew a curve representing its trajectory, which we were able to use to formulate its equation. Applying real-world problems authenticates the curriculum so that it is more meaningful to the students. It gives the equations and symbols a purpose and a reason. Furthermore, students are more apt to retain information if they are able to make a real connection with it; one of my goals is to integrate this pedagogical technique into students’ learning.

Another common complaint voiced by students in mathematics classrooms is that “I’m just not good at math.” This fixed mindset is fallacious and has the power to destroy the potential of a learner. All students have the ability to learn any concept through hard work when provided with the necessary resources and sufficient time. As a teacher, I would like to replace this fixed mindset with a growth mindset that encourages effort over innate talent. Such a mindset is essential to student success and motivation. My goal is to have a classroom based on a growth mindset so that students have the confidence to face challenges as they expand their abilities and reach their full potential in the world of mathematics.

The need for equal learning opportunities for all students along with the importance of mathematics has inspired me to pursue a career as a high school mathematics teacher. I hope to pass on to my students my passion for the subject while encouraging them to take their knowledge further than the classroom and into the real world. Because life without mathematics is … impossible!
After a very busy 2013-14, things were somewhat quieter in the Ph.D. program this year. One student, Elizabeth Drellich, graduated in February, and Jeff Hatley is getting a May degree. In addition, Mei Duanmu, Stephen Oloo, Evan Ray, Peng Wang, and Tobias Wilson are all expected to graduate this summer.

The department granted twenty-eight masters degrees this year: Michael Boratko, Konstantinos Gourgoulias, Andrew Havens, Konstandinos Kotsiopoulos, Dan Nichols, and Cory Ward in mathematics; Orhan Akal, Cassandra DePietro, Janisa Henry, Shaina Rogstad, Courtney Tilley, and Wenlong Wang in applied mathematics; and Kathryn Aloisio, Yuanyuan Chen, Jun He, Jingyao Hou, Jing Hu, Anna Kye, Zhengyang Liu, Domonic Mei, Michael Miller, Ning Ouyang, Danie Paul, Yue Tang, Li Wang, Lap Kan Wong, Yifeng Wu, and Yipeng Yang in statistics.

Congratulations to all these students!

During the summer of 2014, Matthew Bates worked as a teaching assistant for a course in mathematical logic for high school students offered by the Center for Talented Youth in Los Angeles. This summer he will work as junior staff at the Hampshire College Summer School in Mathematics.

In August, Isabelle Beaudry gave a presentation titled “Correcting for preferential recruitment in respondent-driven sampling” at the Joint Statistical Meetings in Seattle.

A number of our students, including Zhijie Dong, Huy Le, Kien Nguyen, Arie Stern, Tassos Vogiannou, and Feifei Xie, attended the fall and/or spring AGNES algebraic geometry conferences at Penn and Boston College.

Three of our graduating Ph.D. students, Elizabeth Drellich, Jeff Hatley, and Stephen Oloo went to the AMS-MAA Joint Mathematics Meetings in San Antonio, Texas in January. Elizabeth gave a talk titled “Valid plane trees: combinatorial models for RNA structures with Watson-Crick pairs” on work with Julianna Tymoczko and Frances Black (Smith BA ’14). Her Smith College research students also gave a talk there titled “Modeling RNA.”

In March, Boqin Sun gave a talk titled “Quantile regression for survival data with delayed entry” at the spring meeting of the Eastern North American Region International Biometric Society in Miami.

This summer, Feifei Xie will attend a graduate summer school in geometric group theory at the Mathematical Sciences Research Institute, Berkeley. His attendance is made possible through the department’s institutional membership with MSRI. Kien Nguyen is attending the Park City Mathematics Institute’s graduate summer school on geometry of moduli spaces and representation theory.

In April, Haitao Xu gave a talk in a minisymposium on the topic “Mathematical progress on nonlinear phenomena in parity-time-symmetric systems” at the 9th IMACS Conference on Nonlinear Evolution Equations and Wave Phenomena in Athens, Georgia.

In November, Zijing Zhang attended the SIAM Conference on Financial Mathematics & Engineering in Chicago.

Distinguished Thesis and Teaching Awards

The Graduate Affairs Committee selected two students for this year’s Distinguished Thesis award, Jeff Hatley and Peng Wang. This award recognizes the student or students who show the most promise for a research career as evidenced by an excellent thesis or paper.

In March Jeff Hatley defended his thesis, in the area of number theory, under the direction of Professor Tom Weston. He proved two separate results about the deformation theory of modular forms, extending Weston’s criterion for unobstructedness of deformations from squarefree
level to arbitrary level. Weston writes that “[t]his is not a small improvement: I stuck to the case of squarefree level because that was all I knew how to do. The possible ramification is vastly simpler in the squarefree level case, and grappling with the possibilities in the general case was a major undertaking.” A paper containing results from Jeff’s thesis has been accepted for publication in the International Journal of Number Theory. In the fall he will begin a three-year Visiting Assistant Professor position at Union College in Schenectady, NY.

Peng Wang defended his thesis in April under the direction of Professor Anna Liu. Professor Liu writes that “Peng’s thesis is motivated by two collaborative projects. One is to uncover the concentration of the dissolved organic carbon at the outlet of a watershed due to different land types (such as forest, wetland and urban area) that constitute the watershed. The other is in the TV rating industry, to estimate viewers’ watching patterns. ... Peng’s innovation lies in 1) using a single index varying coefficient model with variable selection methods to achieve model parsimony and interpretability for relatively high dimensional and nonlinear data; 2) devising a two-step iteration algorithm for model estimation and implementing [it] with an R package. Peng has presented his thesis work in last year’s New England statistics symposium. I anticipate at least one publication in the next year in a good quality journal.” Peng is currently working for Google as a statistician.

The departmental Distinguished Teaching awards were given to Isabelle Beaudry and Tom Shelly. This award recognizes excellence in teaching, with preference given to students who have taught a variety of courses and have contributed in multiple ways to the teaching mission of the department.

Isabelle Beaudry is starting her fifth year in the Statistics Ph.D. program in September 2015. She has done an excellent job leading discussions for Stat 240 four times and teaching Stat 111 in the summer; in the course evaluations students said that Isabelle was well-prepared, thorough, and extremely helpful. In fall 2014 Isabelle was the primary instructor for Math 437, actuarial financial math. This was a great responsibility and a tremendous contribution to the department’s teaching mission. Director of Staff, Ilona Trousdale, writes, “I met with her several times over the semester and I know she put a great deal of thought, care, and work into teaching the course.” Her students rated her teaching very highly, saying she was “helpful,” “easy to talk to,” “welcomed questions,” “always willing to help,” and “very clear and organized.”

Tom Shelly has been primary instructor in a wide range of our undergraduate courses: he taught Calculus I twice, Calculus II once, Calculus III three times, and is now teaching Linear Algebra. He also taught calculus at Smith College during the fall of 2014. Ilona Trousdale writes that he “clearly loves teaching and his evaluations are excellent. Students appreciate his examples and explanations. Tom is very good at engaging undergraduate students with his energy, enthusiasm and humor. As the math club organizer, Tom arranged a nice series of talks and activities for undergraduate math majors this year. He was also co-organizer of the math club with Nico [Aiello] last year.” Tom is continuing in the Ph.D. program in the fall.
at a florist, all so that he could provide a decent living for his family. Whitaker also looked up to his uncle, the first one in his family to go to college, who frequently visited and took an interest in him, asking him math questions.

Whitaker went to segregated schools up until the 8th grade, finishing his secondary education in a predominantly white school. He matriculated at Hampton Institute, a Historically Black College in his hometown. Nate liked math, had been good at it, and chose math as his major. It was a tumultuous time. At Hampton, several semesters were cut short because of student unrest due to civil rights and the Vietnam War. Whitaker changed his major, and he eventually graduated with a degree in economics with a math minor.

After graduation, Whitaker worked for the Army doing cost-benefit analyses on weapon systems and logistics programs in Petersburg, Virginia, about 60 miles from Hampton. Being isolated with lots of free time, he started taking evening classes in physics and math. Nate enjoyed this and did very well. In fact he enjoyed it so much that he decided to quit his job and go to graduate school full time in mathematics. Nate entered the University of Cincinnati in the fall of 1979. This was not an easy decision for him, but he went into it with all that he had, working harder than he had ever worked before. Due to the lack of English speakers in the graduate program he was immediately thrust in front of a classroom of 62 students (56 female), teaching math for elementary school teachers. Having never taught before, Whitaker remembers during the first month that he would only look at the board during his lectures, being too nervous to turn around. Over the semester, he got better and actually grew to enjoy it.

In his first year at Cincinnati, Whitaker took a numerical analysis course from Diego Murio. This was Murio’s first year teaching there, having just finished his PhD at the University of California at Berkeley. Murio lectured with such passion and energy. Nate remembers that Murio would be exhausted at the end of most classes, writing down the last of his lecture on the board while sitting in a chair. Nate loved the course. He especially liked the way that mathematical analysis and computing complemented each other. He worked extremely hard in his classes, especially in Murio’s.

Whitaker planned on getting a job after graduation, but Murio asked him about pursuing a PhD and suggested Berkeley. Whitaker decided to apply to Berkeley and New York University only, assuming that he would probably not get accepted, but he was accepted at both. He completed his master’s degree at the University of Cincinnati and subsequently enrolled at Berkeley in the fall of 1981.

Nate approached Berkeley in the same manner that he had approached Cincinnati: giving it his all. He found Berkeley very different from Virginia and Cincinnati, which both had an air of the old segregated South. Whitaker was the only African-American graduate student at Cincinnati, but at Berkeley there was a critical mass of minority graduate students who were very successful in the graduate program. This was partially due to the Mathematics Opportunity Committee, which provided opportunities for graduate study at Berkeley to under-represented groups through admissions and financial assistance. Whitaker and the other minority students tried to support one another through weekly tutorials designed to prepare for the first basic exam. The minority students organized social gatherings to establish a sense of community. Whitaker believes that this contributed to the success of minorities in the graduate program, noting that Berkeley graduated more African-Americans with PhDs in mathematics than any other university in the 1980s. He is very proud to have been a part of that.

At Berkeley, Whitaker worked on his dissertation under the direction of Alexandre Chorin. He studied numerical algorithms for solving the equations modeling fluid flow between 2 plates – the Hele-Shaw cell – a simple model for flow through porous media. He completed his PhD in 1987 and came directly to UMass Amherst as an Assistant Professor.

Whitaker is a numerical analyst who develops and implements algorithms to solve physical and biological problems described by differential equations. During his time at UMass, he has published articles in several research areas, including 2-dimensional turbulence, blood flow in the kidney, tumor-induced angiogenesis, waves in electro-magnetic fields and Bose-Einstein condensates. Nate says that he loves his job because he feels that he has a license to explore new research areas and to try to make an impact in these areas. His career has opened many unexpected opportunities and broadened his horizons. He has been able to travel to different places around the world to collaborate with scientists on research projects. For example, in 1995,
Whitaker spent the year in Lyon, France, at École Normale Supérieure on sabbatical doing research.

Whitaker also treasures his interaction with undergraduate and graduate students through classroom teaching, research experiences, and advising. He sees a natural beauty in mathematics and tries to convey it to his students. He has supervised 5 PhD dissertations at UMass; three of these students were women, and one was an African-American male. He has also mentored countless undergraduates, many of whom have gone on to be very successful. Nate is particularly proud of a group of 3 students -- Heather Harrington, Marc Maier, and LeSantha Naidoo -- who did a joint honors thesis on tumor angiogenesis with Whitaker and his colleague Professor Panos Kevrekidis. Whitaker had mentored Harrington since she was a freshman and taught her several courses. She won the prestigious Goldwater Scholarship as a junior, earned her PhD in math biology from Imperial College, and is now a junior research fellow at Oxford. Maier went on to earn his PhD in computer science, and Naidoo is a medical doctor.

Whitaker is aware of the lack of opportunities experienced by his parents and by other talented African-Americans before him. It has therefore been important for Nate to make the most of the opportunities given to him and to help and mentor others. For example, between 2004 and 2011, Nate was part of a Saturday morning math program at UMass designed to increase the number of African-Americans students in the honors math classes at Amherst high school. Almost all of the students who started the program in the 4th and 5th grades went on to take AP calculus at the high school. For his work in this program, Whitaker was awarded the University Distinguished Community Service Award as well as the UMass President's Award for Public Service.

Nate Whitaker says that he has the best job in the world. He loves the beauty of mathematics and its connections to other areas. He cherishes his interactions with his students and the possibility of making an impact in their lives. His approach to life is based on principles that he learned from his parents. One of the guiding principles that his mother imparted on him is “To whom much is given, much is expected.” Nate feels that he has been fortunate in life, and he continues to look for ways to make a difference.
Nestor Guillen joined the department as an assistant professor in September 2014. He works on the analysis of partial differential equations (PDEs), with a focus on an important class of PDEs known as nonlinear elliptic equations. These nonlinear equations appear in many areas of mathematics and the sciences. For example, both the problem of constructing a surface with prescribed curvature and the problem of constructing a surface so that light beams reflect from it in a desired fashion lead to a nonlinear PDE known as the Monge-Ampere equation. Nestor’s research is inspired, in part, by questions involving the thermal and electrical properties of composite materials, which involve global properties of heterogeneous geometry. The study of these questions blends ideas from harmonic analysis, Riemannian geometry, and stochastic processes.

Nestor, born and raised in the Venezuelan Andes, moved to the United States in 2006 to start his Ph.D. studies at the University of Texas at Austin, where he was mentored by Professor Luis Caffarelli. Before coming to UMass Amherst, he was an E. R. Hedrick Assistant Professor at UCLA, where Inwon Kim was his postdoctoral mentor. During the spring semester of 2011 he was a fellow at the Mathematics Sciences Research Institute at the University of California at Berkeley, and during the fall semester of 2014 he had a visiting position at the Fields Institute at the University of Toronto, where he was a member of a semester long-program on Optimal Transport, Geometry and Calculus of Variations.

Lisa Bergman now provides the department’s duplicating service, returning to the university following a long hiatus. She most recently worked for the Department of Psychology and the Provost’s Office as a database programmer. She also worked for many years as a technician, primarily for environmental biology studies. Lisa studied microbiology and music at Hampshire College. She has two sons, the younger of whom is a freshman engineering student at UMass Amherst.

John Folliard provides support and systems administration as part of the Research Computing Facility in the department. His prior experience includes support and consulting for higher education and small businesses. John enjoys music, the ocean, and traveling, as often as possible with his wife, Jenna. Milo, their rat terrier, celebrated his tenth birthday in April.

Cathy Russell joined the department this past fall as assistant to the Birkhoff Professor, Bill Meeks. In addition Cathy provides administrative and academic support to the department. Previously she worked in the Department of Polymer Science and Engineering at UMass Amherst as manager of the VISUAL Program. In her free time Cathy enjoys weaving, and during the past fall she received her Master Weavers Certificate. On weekends she enjoys working as a real estate agent.

Kam Kit Wong is the Travel Preparer and the Assistant to the Director of Applied Mathematics in the department. She grew up in Hong Kong, and her previous marriage brought her to the U.S. Before joining the university, Kam worked in payroll at the Evaluation Systems Group of Pearson in Hadley for the past 16 years. She has one daughter, who will be a student at Amherst High School next year.
DONALD GEMAN: ELECTION TO THE NATIONAL ACADEMY OF SCIENCES AND HIS FAMOUS 1984 PAPER

by Joseph Horowitz

Donald Geman, a faculty member in our department from 1970 to 2001, was elected to the National Academy of Sciences (NAS) in April “in recognition of [his] distinguished and continuing achievements in original research.” Don is currently in the Department of Applied Mathematics and Statistics at Johns Hopkins University.

During his first fifteen years at UMass Amherst, Don and I collaborated on a series of theoretical papers on stochastic processes, but by 1980 our interests had shifted to more applied questions. Around that time Don established the Statistical Consulting Center in the department and was the director until 1984. Meanwhile, on the research side, he had branched out and begun working on problems in image analysis with his brother Stuart (elected to the NAS in 2011) and others at Brown University.

In 1984, the brothers Geman published one of the landmark papers in statistics, “Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images” (IEEE Transactions on Pattern Analysis and Machine Intelligence, 6), referred to as G2 below. I shall try to explain the essence and significance of G2 in a nontechnical manner, keeping in mind the dictum, ascribed to Einstein, that everything should be made as simple as possible, but no simpler, and Stephen Hawking’s admonition in his book A Brief History of Time that the inclusion of “each equation … would halve the sales” of the book.

G2 is ostensibly about the restoration of digital images that have been degraded by noise, blur, or other distortions. Digital images, now common in cameras and phones, were much less familiar in 1984. A famous example of degraded images is provided by those from the Hubble Space Telescope before its repair. Earlier approaches to image analysis included statistical and ad hoc methods, but it is fair to say that in 1984 the theory and practice were in a rather primitive state.

The results of G2 consist of three interrelated components. The first is an image model comprising a prior distribution, a likelihood function, and a posterior distribution; in short, a Bayesian statistical model. The prior distribution describes probabilistically the population of possible images from which the original nondegraded, “pristine” image is assumed to have been chosen; its specification is all important. The prior used by the Gemans is a Markov Random Field (MRF), a 2-dimensional random process with the property that, for predicting the state, or color, of any particular pixel, only the neighboring pixels contain any relevant information. Which pixels are to be considered “neighbors” and the strengths of the “bonds” between neighbors, which measure the degree to which the states of neighboring pixels affect each other, are part of the MRF definition. One can imagine how properties of actual images, such as smoothness or choppiness, might be encoded in such specifications, and this makes MRFs ideal as models for images.

The likelihood function specifies how distortions, such as random noise, change the statistical structure of the original image. Which distortions are possible depends on the situation; for example, film photography and the Hubble Telescope are afflicted by different types of noise and distortions. Action of the distortions on the pristine image yields the observed image, the actual datum of the problem, and leads to the posterior distribution, which is the modified probabilistic description of the population of pristine images in light of the data. Restoration of the original image consists in choosing the image(s) deemed most probable by the posterior distribution. This is a difficult maximization problem not easily addressed directly in the MRF framework.

Aside from their flexibility and intuitive appeal as image models, MRFs have the property that they can always be represented as Gibbs distributions, a fact not noted by earlier workers in this area. A Gibbs distribution is a particular type of probability distribution that first arose in physics; its mathematical representation incorporates the neighborhood structure and bond strengths of the MRF model into a formula that corresponds to energy in the physics context. The prior Gibbs distribution gives the probability, before looking at the data, that any particular pristine image is the one that has been chosen. What is crucial and what leads to the second component of G2 is
that the posterior distribution is also a Gibbs distribution.

In 1953 Metropolis et al (J. Chem. Physics, 21) employed Monte Carlo methods to compute the minimum energy state of a physical system consisting of a large number of particles, using the ancient MANIAC computer at Los Alamos on a system of 224 particles. The Geman noted that maximizing the posterior distribution was equivalent to minimizing the energy term in the Gibbs formula. Modifying the method of Metropolis et al they showed how to implement the necessary large-scale computer calculations, dubbing their algorithm the Gibbs sampler. The Gibbs sampler takes an initial “guess” at the original image and, in conjunction with a technical modification, called simulated annealing, alters it in a sequence of simple steps designed ultimately to minimize the energy term. This generates a random, nonstationary Markov process whose states are the succession of computed images. The third component of G2 is a mathematical proof that, if allowed to run indefinitely, this Markov process would actually arrive at the original image. In practice, of course, the algorithm is stopped after a finite number of steps.

Thus G2 provided a flexible, computationally feasible statistical model, indicated how to perform the computations, and gave mathematical results ensuring that the method would work. The paper provoked much follow-up work in image analysis, but there is an equally important statistical sequel to the story that will clarify the word ‘ostensibly’ used above.

For years statisticians have debated the relative merits of Bayesian and frequentist statistics, sometimes with religious fervor. Many of them might have agreed that Bayesian is theoretically or philosophically superior to frequentist analysis, but in the 1980s Bayesian computations were hardly feasible except in relatively simple situations. Although it was introduced for image analysis, A. Gelfand and A.F.M. Smith (J. Amer. Stat. Assoc. 85, 1990) recognized that the Gibbs sampler could be used for many other optimization problems. They showed how to adapt it and other Monte Carlo methods to solve numerical problems in statistics. This unleashed a flood of work, under the rubric Markov chain Monte Carlo, that allowed statisticians to attack many problems that were not approachable earlier, a development that perhaps now overshadows the original contribution of G2.

The papers by Geman and Geman, Gelfand and Smith, and W.K. Hastings, who introduced similar methods into statistics (Biometrika 87, 1970), were recognized in Breakthroughs in Statistics Volume III (S. Kotz and N.L. Johnson, 1997). In 2005 G2 was listed as one of the 25 most cited papers in statistics (T.P. Ryan and W.H. Woodall, J. Appl. Stat. 32). Google Scholar mentions 16979 total citations, currently about 1000 per year, more than most mathematics and statistics papers garner in a lifetime.

In addition to this early work, Don has made significant contributions to mathematics, statistics, computer vision, machine learning, and, most recently, computational medicine.

In preparing this article I took the opportunity to reread G2. It is beautifully written and still worth reading after more than 30 years. I also had the benefit of conversations with Don Geman thirty-some years ago as well as more recently.

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**RECENT FACULTY PROMOTIONS**

At its meeting on June 17, 2015, the Board of Trustees approved recommendations from Provost Katherine Newman and Chancellor Kumble Subbaswamy to promote three members of the Department.

*Daeyoung Kim* joined the Department as Assistant Professor in September 2008, having done his graduate work at Penn State. His work in statistics spans topological statistics and mixture models.

*Alexei Oblomkov* obtained his PhD at MIT, then did postdoctoral work at Princeton prior to joining our faculty in 2009 as Assistant Professor. He studies representation theory, algebraic geometry, and low-dimensional topology, especially the interactions between these fields. We are very pleased that Kim and Oblomkov have now joined the tenured faculty at the rank of Associate Professor. *Siman Wong* joined the Department as Assistant Professor in 1999, following graduate work at MIT and postdoctoral work at Brown. A past Director of the Graduate Program, he now serves as the Undergraduate Program Director. His research specializes in analytic as well as algebraic number theory. By vote of the Board of Trustees, he will start the fall term at the rank of Professor.

Congratulations to all three for these well-deserved promotions!
ALUMNA PROFILE: JOAN BARKSDALE '66

Each year the newsletter selects an outstanding graduate or faculty member from our program and profiles their career. This year we were privileged to interview Joan Barksdale.

Joan was an undergraduate at UMass Amherst from 1962 through 1966, and she has gone on to serve the University in many ways since that time, most recently as the Chair of the College of Natural Sciences (CNS) Campaign Council. The interviewer joined the faculty here in 1967, so we have many colleagues and experiences in common.

Newsletter: Tell us about your early years and about your experiences as an undergraduate here at the University of Massachusetts Amherst.

Barksdale: My family lived in Rhode Island. When I was about to attend high school, we moved to Seekonk, Massachusetts, on the border with Rhode Island. Seekonk, at that time, did not have its own high school, so I attended high school in East Providence, R.I. Upon graduation and looking on to college, I needed a school which would offer a quality education at a price which my family could afford. In Massachusetts the obvious choice was our University in Amherst. The University was expanding at that time in all academic directions because the times demanded it. Remember, this was a time when President Kennedy stated boldly that we would put a man on the moon within a decade.

I didn't originally come to study mathematics, but history. Professor William Davis of the Department of History was my advisor. In consultation with him, he said to me, "I see you have done very well in mathematics, and there are not many women today who pursue a career in that field. This would offer you a great opportunity if you were to choose this field." He was a wonderful teacher and had the concern and foresight for his students, and it was under his advisement that I began my serious study of mathematics.

Let me tell you about some of my teachers. It is difficult to remember their names after all these years. There were a few women in the department at this time, but one stands out in my memory, Professor Eleanor Killam. She was tough, demanding and, most importantly, very good. She was certainly a model of achievement for a young woman to see. About halfway through my studies Professor Wayman Strother became chairman of the department. While I didn't study with him, he became instrumental in my future career. The Federal Government was offering graduate fellowships for teachers of mathematics in a program developed by the U.S. Office of Education. One of the program sites was the University of Pennsylvania. It was Professor Strother who helped me secure this fellowship, which resulted in attaining my M.A. degree in Mathematics Education Research.

Newsletter: Tell us about the years after the University of Pennsylvania.

Barksdale: Ed and I married while we were at Penn, then moved all over the United States. I taught mathematics, statistics, and mathematics for teachers at DeKalb County Community College in Georgia, Monmouth College in New Jersey, and Golden Gate University in California. On our return to Connecticut I took time off to raise a family and then became involved in philanthropic efforts to encourage students to pursue higher education.

Newsletter: We mentioned earlier that you are the chair of the CNS Campaign Council. Please tell us about this and any other philanthropic activities you are involved in.

Barksdale: The CNS Campaign is the fundraising activity for the College of Natural Sciences under the UMASS RISING Campaign for the University of Massachusetts Amherst. The CNS goal for the campaign is $77 million out of the $300 million total for the University. Many people believe, since we are a state institution, that taxes automatically support the University. In fact, state funding amounts to less than 50% of the costs incurred each year. It makes sense for the University to go after private philanthropy to fill the gap left by the shortfall of both state and federal funding in all areas of university need: faculty and student support, physical plant, research efforts, core programs, and endowment. I hope that all of our readers will consider making a gift to this effort.

Newsletter: Let me interject. I know as a member of the Department of Mathematics and Statistics how fundamental your personal support of our Research Experience for Undergraduates (REU) program has been to our summer student activities. You may be out campaigning for funding, but you are also a direct contributor to our program, and we thank you very much for this!

Barksdale: I am also the current scholarship chair of the Norwalk Community College Foundation in Norwalk, Connecticut. While this is fundraising at a different level than the CNS efforts, it is so worthwhile for those who need a close-to-home and more affordable alternative to higher education.

Newsletter: Joan, it has been a delight and a privilege to chat with you about your past and present connections to UMass Amherst. We all hope you are successful with your future philanthropic efforts, and we appreciate all of your gifts and work on the University's behalf. Thank you again!
The following alumni and friends have made generous contributions to the Department of Mathematics and Statistics between January 2014 and June 2015. Your gifts help us improve our programs and enrich the educational experiences of our students. We deeply appreciate your continuing support.

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HONGKUN ZHANG NAMED 2015 SIMONS FELLOW

Professor Hongkun Zhang has been awarded a 2015 Simons Fellowship for her project titled “Stochastic Perspectives of Billiard Dynamics.” The awardees for 2015 are announced on the Simons Foundation website https://www.simonsfoundation.org/mathematics-and-physical-science/. The Simons Foundation, established by noted mathematician and investor James H. Simons, awards fellowships annually to leading researchers in mathematics, theoretical physics, neuroscience, and other fields. The foundation awards a maximum of 40 fellowships in mathematics annually to faculty in the U.S., Canada, and the United Kingdom. The Fellows Program under the Simons Foundation Division for Mathematics and the Physical Sciences provides funds to faculty for up to a semester-long research leave from classroom teaching and administrative obligations. Simons Fellows are chosen based on research accomplishment in the five years prior to application and the potential scientific impact of the fellowship.

While supported by the Simons Fellowship during 2015–2016, Zhang plans to investigate mathematical problems arising in equilibrium and nonequilibrium statistical mechanics, particularly in the context of billiard dynamics, gases, and diffusion phenomena. She plans to seek both a theoretical understanding as well as new ways to connect mathematical ideas to a variety of complex phenomena by investigating the statistical properties of physical systems and their mathematical models. One of the tools that Zhang will use is an innovative coupling technique for nonuniformly hyperbolic systems that she has developed and that represents a significant advance in the field.

An important model of billiard dynamics was constructed in 1970 by Yakov Sinai, who in 2014 was awarded the prestigious Abel Prize. This model, which relates to a mathematically tractable model of a gas, has produced a renewed convergence of interests between physicists and mathematicians working on the theory of chaotic, hyperbolic dynamical systems. One of the links between such systems and probability can be seen if the dynamical system \((T, M, \mu)\) is (strongly) mixing. In this case, for any random variable \(f\), the sequence \(X_n = f \circ T^n, n \geq 0\), defines a stationary stochastic process on the probability space \((M, \mu)\). The random variables \(\{X_n\}\) are usually not independent; in fact, they are typically highly correlated. Nevertheless this observation allows us to use the theory of stochastic processes to study the deterministic dynamical system.

The rate of decay of correlations of stochastic processes generated by the flows \(X_n = f \circ T^n, n \geq 0\), plays a key role in applications to physical systems. Although exponential decay of correlations for the Sinai billiard map was proved by Young and Chernov, the rate of decay of correlations for the billiard flow is a much harder problem that remains unsolved. Zhang plans to investigate the rate of decay of correlations for processes generated by billiard flows together with Sandro Vaienti in Marseille during the fall of 2015.

Zhang plans to continue her research work in random billiards to demonstrate the power of mathematical reasoning in understanding the natural world using the theoretical results and newly developed tools in chaotic billiards. Together with Renato Feres of Washington University in St. Louis, she will explore new model systems that are of interest to applied scientists and engineers, such as Knudsen diffusion in nano-structured channels and thermo-mechanical behavior of nano-devices.