When the Newsletter editors asked me if I would write something about my career in the Department, and especially my plans for the future, I was initially reluctant, sensing such a testimonial might seem too self-absorbed. But then it occurred to me that it might also be a good way to communicate with all the students, colleagues and staff I have known over the past 33 years, who are readers of this Newsletter. Over the past few months, whenever I have met someone who asks “What are you going to do once you retire?”, it is usually in the elevator, and knowing that I have only 15 seconds before the door will open and my questioner will depart, my answer is all too brief. Since the question is invariably asked with genuine interest, it deserves a proper answer. So let me try here.

First a little history. There is no doubt in my mind that being hired into the Department in 1984 was the luckiest break of my career. Immediately I felt at home with my colleagues, and soon I fell into some productive collaborations. George Knightly and Mel Berger were running the Center for Applied and Computational Mathematics, which was then, as it is now, the nexus for a group

EQUITY, INCLUSION, AND DIVERSITY
In December 2016, a group of department members formed a voluntary ad hoc diversity committee, in the interest of amplifying and multiplying the messages of welcome and inclusiveness in the Department. We are lucky to be in a department with great collective interest in supporting a welcoming atmosphere for people from many backgrounds. On 23 February 2017 the faculty adopted this departmental diversity statement by unanimous vote:

“The Department of Mathematics and Statistics includes and serves Faculty, Staff, and Students from a wide range of personal and professional backgrounds. It has long been a pillar of academic thought that diversity of perspectives fosters enhanced ability to advance science. We know that having a department of teachers, mentors, staff, researchers, and colleagues from a range of personal and professional backgrounds also enhances our ability to effectively serve our educational mission. We embrace the diversity of our department and community, including persons of varying age, disability, ethnicity, family status, gender, gender identity, geographic region, national origin, political affiliation, race, religion, sexual orientation, and socioeconomic status.”

FACULTY PROFILE: REFLECTIONS UPON MY RETIREMENT
When the Newsletter editors asked me if I would write something about my career in the Department, and especially my plans for the future, I was initially reluctant, sensing such a testimonial might seem too self-absorbed. But then it occurred to me that it might also be a good way to communicate with all the students, colleagues and staff I have known over the past 33 years who are readers of this Newsletter. Over the past few months, whenever I have met someone who asks “What are you going to do once you retire?”, it is usually in the elevator, and knowing that I have only 15 seconds before the door will open and my questioner will depart, my answer is all too brief. Since the question is invariably asked with genuine interest, it deserves a proper answer. So let me try here.

First a little history. There is no doubt in my mind that being hired into the Department in 1984 was the luckiest break of my career. Immediately I felt at home with my colleagues, and soon I fell into some productive collaborations. George Knightly and Mel Berger were running the Center for Applied and Computational Mathematics, which was then, as it is now, the nexus for a group

Article continues on page 27
NEW CHALLENGE PROBLEMS

Here is a selection of some of the more challenging problems from the 2017 Jacob-Cohen-Killam prize exam. Three additional prize exam problems are included in the Awards Dinner article on page 18.

Problem 1. You start with an empty bucket. Every second, you either add a stone to the bucket or remove a stone from the bucket, each with probability 1/2. If you want to remove a stone from the bucket and the bucket is currently empty, you do nothing for that second (still with probability 1/2). What is the probability that after 2017 seconds the bucket contains exactly 1337 stones?

Problem 2. Suppose you are standing in a field near a straight section of railroad tracks, just as the front of the train locomotive passes the point on the tracks nearest to you, which is 1/4 mile away. The train is 1/3 mile long and traveling at 20 mph. How slowly can you run and still catch the train? In what direction should you run?

Problem 3. Find all real solutions of the equation
\[ 2(2y-1)^{1/3} = y^3 + 1. \]

Problem 4. Suppose there is a circular road with \( N \) gas stations having in total just enough gas for your car to go around the circle once. Your car’s gas tank is empty. Prove that there is some gas station at which you can start so that you make it around the circle without running out of gas.

Problem 5. Show that for any positive integer \( m \)
\[ m^{2017} = \sum_{k=0}^{2017} \binom{m}{k} x_k, \]
with integers \( x_k \). Here we set \( \binom{m}{k} \) = 0 if \( k > m \).

Problem 6. Through the center of a cube draw a straight line in such a way that the sum of the squares of its distances from the vertices is (a) minimal, (b) maximal.

Although he is on sabbatical in Berlin this year, you may still submit your solutions to the Problem Master, Professor Franz Pedit, via email <pedit@math.umass.edu>.

Last year’s solutions are on page 20.
DEPARTMENT HEAD’S MESSAGE

The Department of Mathematics & Statistics continues to grow and renew itself. The number of mathematics majors grew to 857 this year, meaning it has just about doubled in the last four years. The most popular concentrations are Actuarial, Computational Math, Statistics, and Applied Math. To meet the increased demand, the Department will need to continue to add more staff at all levels. This year, we matched the pace of hiring from the remarkable period around fifteen years ago. We hired a total of 14 new faculty members (4 Assistant Professors, 8 Visiting Assistant Professors and 2 Lecturers). The need for more space has never been clearer, and appears to be becoming more evident to the university administration: already this year we have added a suite of offices for Lecturers on the first floor. I have every expectation that next year’s Newsletter will contain news of a much more substantial investment in renewal and expansion of space for Mathematics & Statistics in Lederle Tower.

In last year’s Newsletter, this column hinted at an upgrade in our VAP program which I am extremely pleased to report is now a reality. Namely, the Department has created the new Marshall H. Stone Visiting Assistant Professor position in honor of a great American mathematician with a close connection to Amherst and to our Department. Marshall Harvey Stone (1903-1989) was one of the most celebrated mathematicians of the 20th century. He made lasting contributions to Functional Analysis, particularly Spectral Theory, to Topology, and to Logic. His many accomplishments include the Stone-Weierstrass Theorem about the approximation of functions by polynomials, the Stone-Čech compactification of a topological space, and Stone’s representation theorem for Boolean algebras, all key results in diverse areas of mathematics. Stone received his PhD from Harvard University in 1926 where he returned as a Full Professor in 1937. In 1946 he became Chair of the Mathematics Department at the University of Chicago and transformed it into a major center of US mathematics, recruiting André Weil among many other important mathematicians.

In 1968, Marshall Stone retired from Chicago and joined the Department of Mathematics & Statistics at the University of Massachusetts Amherst as the inaugural George David Birkhoff Professor of Mathematics. He was a member of the National Academy of Sciences, and president of both the American Mathematical Society and the International Mathematical Union. Professor Stone was distinguished with the National Medal of Science in 1983. His connections with Amherst run very deep. His father, Harlan Fisk Stone, a graduate of Amherst High School and Amherst College, was named a Justice of the U.S. Supreme Court in 1925, and served as Chief Justice from 1941 to 1946.

The Department is very pleased to announce the inaugural Marshall H. Stone Visiting Assistant Professors are Jeremiah Birrell, Taryn Flock and Luca Schaffler. The other VAPs appointed this year are Mareike Haberichter, Eric Hall, Noriyuki Hamada, Zuhair Mullath, Dinakar Muthiah and Anna Puskas. In the Lecturer search, we are very pleased to have hired Michael Hayes and Maria Nikolaou. The total number of VAP positions in the department will increase by an additional two next year, thanks to Strategic Investment funds we garnered. The same funds will also allow us to add a new staff member for professional academic and career advising later this year, a significant addition. Speaking of staff, this year we welcomed a new Business Manager, Christine Curtis.

Turning to hiring at the tenure track level, we succeeded in recruiting four talented Assistant Professors: Owen Gwilliam, Vincent Lyzinski, Alejandro Morales and Annie Raymond. These additions represent significant strengthening of our groups in Algebraic Geometry, Statistics and Data Science, as well as the creation of a new group in Discrete Mathematics. Raymond and Gwilliam will start their appointments in January and September of 2018, respectively. Since my space here is limited, I invite you to look for a profile of each of these new additions to our Department in next year’s Newsletter.

I end this year’s message on a bittersweet note, as in a few days I will be leaving my current post to serve as the Senior Vice Provost for Academic Affairs. I am absolutely convinced that the Department is on a solid path toward rising in the quality of its teaching, research and outreach efforts which will bring it ever greater renown and visibility. As I undertake a University-wide administrative role, my home department will continue to occupy a special place in my vision for the tremendous potential represented by the faculty, staff and students of the flagship University of Massachusetts.

– Farshid Hajir
ANDREW HAVENS EXPLORES THE MATHEMATICAL THEORY OF BRAIDS

The term *braid* typically evokes hair or rope, rather than diffeomorphisms of multiply-punctured disks or loops in a configuration space — or that proprietary algebraic sauce that someday may secretly and securely authenticate pairings between electronic devices. All of these notions are woven together in the mathematical theory of braids. Among mathematicians, the term refers both to a geometric object resembling intertwined strands of rope, and also to an element of an algebraic object — the *braid group* — whose combinatorial description captures the topological behavior of a geometric braid.

Underlying the powerful abstraction of the braid group is an algebraic condition, known as the *braid relation*, which captures something simple and well known to knot theorists, low-dimensional topologists, algebraic geometers, and non-abelian cryptographers alike. This relation appears in many mathematical guises and — like Euler’s identity\(^1\) — captures a deep mathematical truth, yet it becomes obvious when the appropriate pictures accompany the symbols.

This illustrated article provides a glimpse into the theory of braids and explores the braid relation from several vantages. We begin with a geometric picture, adopt a topological one, discover an algebraic perspective, and then see these coalesce in the language of the *mapping class group*.

We define a *geometric braid\(^2\)* first. Fix a positive integer \(n\) (to later make sense of the braid relation, it suffices to imagine \(n = 3\)). Denote the closed unit complex disk by

\[
D = \{z \in \mathbb{C} : |z| \leq 1\}.
\]

Let \(P\) be the \(n\) real points equally spaced on the interior of the disk:

\[
P = \left\{ \frac{2j - n - 1}{n + 1} : j = 1, \ldots, n \right\} \subset D.
\]

Now consider a solid cylinder \(D \times [0, 1]\), and let \(P_o\) be the points of \(P\) marked in the bottom disk \(D \times \{0\}\), and \(P_t\) be the points of \(P\) marked in the top disk \(D \times \{1\}\). By a geometric braid with \(n\) strands, we mean a collection of \(n\) disjoint smooth arcs in this cylinder, leaving from \(P_o\) and ending in \(P_t\), such that each arc intersects any slice \(D \times \{t\}\) in exactly one point. This last condition prevents strands from “backtracking” inside the cylinder; equivalently, projecting an arc to its third coordinate defines an increasing function on the interval \([0, 1]\).

Our first definition is too fine to handle easily, so we will coarsen the collection of objects within our grasp: we allow strands to be “wiggled” inside the solid cylinder, without backtracking or passing through each other, and we identify as equivalent any geometric braids that can be transformed into each other by such an isotopy. Reducing to *isotopy* classes of geometric braids shifts us from the wild geometric universe of all permissible disjoint collections of smooth arcs into the more tractable universe of braid classes, since equivalence up to isotopy captures the essential topological relations among the strands of a braid. This isotopy class of a geometric braid will be referred to as a *topological braid*, or simply a *braid*.

With a little more effort, we can turn braids from topological objects into algebraic objects with a combinatorial description via simple diagrams.

We make topological braids into a group by defining the product of two braids via the concatenation operation: glue the cylinders of each braid by identifying the end disk of the first to the beginning disk of the next, then “squish” the resulting height-2 cylinder down to unit height, as in Figure 1. The identity element of this group consists of the isotopy class represented

\[e^{i\theta} = \cos(\theta) + i\sin(\theta)\]

Figure 1. A pair of 5-stranded geometric braids and their product. In this and subsequent figures “~” denotes an isotopy. Can you find a sequence of simple diagrammatic moves realizing this isotopy?

\(^1\)The depth of Euler’s identity \(e^{i\pi} + 1 = 0\) (beyond surprisingly containing the “fundamental” constants in a single equation) lies primarily in understanding complex multiplication by \(e^{i\theta}\) as rotation and complex addition as planar translation; the identity is then an algebraic restatement of the geometric observation that “go halfway around the circumference of a circle, then the circle’s center is halfway between where you are and where you started.” Rotation by \(\pi\) also plays a role in our picture of the braid relation.

\(^2\)Similar definitions can be found in Kamada’s *Braid and knot theory in dimension four*. A convention common elsewhere, such as in Kassel and Turaev’s *Braid groups*, is to use the infinite plane instead of a disk; we opt for the disk to more easily connect geometric braids visually to mapping classes of punctured disks later.
by straight line segments connecting the points of \( P_0 \) to the corresponding points of \( P_1 \) with the same disk coordinate. And the group-inverse of a braid is just its mirror image with respect to the middle disk \( D \times \{ \frac{1}{2} \} \) in the cylinder, which “unbraids” the original braid. In this way, for each \( n \), we obtain an infinite group called the braid group on \( n \) strands.

Here is a diagram illustrating Artin’s generators for the braid group:

![Artin's generators for the braid group](image)

Figure 2. Artin’s generators for the braid group are given diagrammatically by single-crossing projections.

One may suspect that the winding behavior of the strands in a braid can be described combinatorially — this presentation of the braid group was first given concretely by Emil Artin in 1925. To arrive at Artin’s presentation from our notion of topological braids, number the strands 1 through \( n \), and track how the strands pass over or under each other in a generic projection to a plane. From the smoothness and non-backtracking conditions, there is a braid projection with only finitely many crossings where the two strands project to double-points. In such a projection, we can indicate which strand crosses over and which under by breaking the under-crossing strand near the double point.

Our projections can be arranged to give diagrams where \( t \) is a horizontal coordinate, and we may perform an isotopy so at most one crossing occurs for each \( t \in (0, 1) \). Thus we can assume all of the “crossing times” are distinct, and we can “factor” our braids into simpler braids with one crossing each: these are Artin’s generators for the braid group.

For \( n \) strands, there are \( 2(n - 1) \) possible single-crossing behaviors, corresponding to choosing a pair of adjacent strands, and deciding which strand crosses over which. Artin’s generators are then the braids corresponding to single-crossing diagrams where the lower-numbered strand passes over its successor. This gives \( n - 1 \) generators, labeled \( \sigma_1, \ldots, \sigma_n \), where \( \sigma_j \) is the braid whose \( j \)-th strand passes over the \((j + 1)\)-th strand. The inverse \( \sigma_j^{-1} \) of \( \sigma_j \) has the crossing reversed, and the triviality of the braid \( \sigma_j \sigma_j^{-1} \) is a consequence of the fact that the knot theorists’ Reidemeister II move corresponds to an isotopy, a simple change of projection, or both. It’s also clear that two generators commute if the two pairs of adjacent strands they involve are far away from each other: this involves only modifying the isotopy that ensured different crossings occurred at distinct times.

Finally, we introduce the braid relation as the following algebraic identity which holds for any \( j \in \{1, \ldots, n - 2\} \):

\[
\sigma_j \sigma_{j+1} \sigma_j = \sigma_{j+1} \sigma_j \sigma_{j+1}.
\]

Expressed in diagrams as in Figure 3, observe that the braid relation corresponds to two possible configurations of three strands which arise when resolving a triple-point into three double-points. The transformation from one diagram to the other is the knot theorists’ Reidemeister III move. Sliding the middle strand up or down, one can imagine how to lift from a movie of diagrams to an isotopy of the strands in 3-space.

As a consequence of the braid relation, any braid may be reduced to the trivial braid via a sequence of Reidemeister moves. Indeed, the braid group can be generated by the Artin generators and the braid relation. The braid group on \( n \) strands is isomorphic to the free group on \( n \) generators with the presentation:

\[
B_n = \langle \sigma_1, \ldots, \sigma_n \mid \sigma_j \sigma_{j+1} \sigma_j = \sigma_{j+1} \sigma_j \sigma_{j+1} \rangle.
\]

Braids are intimately connected to knots and links. Indeed, closing a geometric braid by gluing together its ends, one obtains a link or knot inside a solid torus. Embedding this solid torus in a standard (unknotted) way into 3-dimensional space, we may regard the closed braid as a classical knot or link in \( \mathbb{R}^3 \) (or the 3-dimensional sphere \( \mathbb{S}^3 \)), as in Figure 5. In fact, Alexander’s Theorem is the well known result that any knot or link in 3-space can be represented as a braid closure. Certain braid closures arise in \( \mathbb{S}^3 \) as the links of singularities of complex algebraic curves in \( \mathbb{C}^2 \).
Here is another way to arrive at the braid group. Consider again the disk $D$ together with the $n$ marked points $P$. We wish to understand smooth, orientation-preserving diffeomorphisms — smooth self-maps of the disk that fix $P$ set-wise, whose inverses are also smooth — up to isotopy. Here, isotopy is now about “wigging” diffeomorphisms: an isotopy between $f$ and $g$, means they are joined by a smooth family of diffeomorphisms. Our old strand-wigging isotopies of geometric braids arise from isotopies of maps, once we understand the strands as the images of maps of the interval $[0, 1]$ into the solid cylinder, with specified endpoints in $P_0$ and $P_1$. By convention, our diffeomorphisms are isotopic to the identity on the boundary (for a disk we can always arrange this), but they are permitted to permute punctures.

More generally, isotopy is an equivalence relation on the group of diffeomorphisms of a space, with composition as the product; the set of isotopy equivalence classes forms a natural quotient group, called the mapping class group of that space. The mapping class group of $D - P$ is exactly the sort of thing we wish to consider: it is just the braid group on $n$ strands! Though a proof of this is beyond the scope of this article, it is not hard to see how a mapping class leaving $P$ invariant contains enough information to specify a topological braid, and with a little work, the reader may convince herself that a topological braid also describes a mapping class. Figure 5 hints at this relation.

The algebraic description of braid groups relies on generating the braid group via half twists of the strands. Arguments in the theory of mapping class groups allow one to generate the mapping class group of the $n$-punctured disk by half-twists along arcs, as illustrated in Figure 6.

To understand half-twists, imagine cutting out a small (topological) disk neighborhood of an arc in $D - P$ and gluing it back in with a rotation by $\pi$ so that the endpoints of the arc are swapped and the arc maps to itself with reversed orientation. Such diffeomorphisms applied to arcs connecting points in $P$ will preserve $P$ as a set, but are not isotopic to the identity through maps that do not alter $P$. Observe however that they are isotopic to the identity if we allow the points of $P$ to move in the interior of the disk. In fact, the graph of the motion of the points of $P$ under such an isotopy, viewed as arcs in the solid cylinder $D \times [0, 1]$, gives a geometric braid.

Figure 5. An illustration of the braid closures for a 3-strand “full twist” and a “standard” 3-strand braid. The former gives a (3, 3)-torus link, while the latter gives the famous Borromean rings. The Borromean rings have the Brunnian property: if any one ring is removed, the remaining pair is unlinked. In contrast, any of the pairs of components in the (3,3)-torus link forms a Hopf link.

Figure 6. The geometric braid associated to an Artin generator corresponds to a right-handed half-twist $H_\alpha$ about an arc $\alpha$ between punctures in a punctured disk.

These isotopies interpolate between diffeomorphisms of the disk that permute the marked points. This leads to another interpretation: a geometric braid is a loop in the configuration space of unordered points on the disk, and the braid group
is the fundamental group — the groups of (based) loops up to homotopy — of this configuration space. (See Kamada’s *Braid and knot theory in dimension four* or Chapter 9 of Farb’s and Margalit’s *Primer on Mapping Class Groups* for a good discussion of this viewpoint.)

Since the braid relation requires three adjacent strands, restrict attention to the braid group on \( n = 3 \) strands. Imagine the disk with 3 punctures at \( P = \{-1/2, 0, 1/2\} \). Label by \( \alpha \) the segment between \(-1/2 \) and 0 along the real diameter, and by \( \beta \) the segment from 0 to \( 1/2 \). Let \( H_\alpha \) denote the mapping class of a positive half-twist along the arc \( \alpha \), and \( H_\beta \) the mapping class of a positive half twist along \( \beta \). Then the braid relation in the mapping class group of \( D - P \) reads

\[
H_\alpha \circ H_\beta \circ H_\alpha = H_\beta \circ H_\alpha \circ H_\beta.
\]

Figure 7 shows the braid relation in this context. From it we see that the compositions of the left-hand-side and right-hand-side of the equation each describe a positive half twist along the arc \( \alpha \cup \beta \). Correspondingly, the braid words \( \sigma_1 \sigma_2 \sigma_1 \) and \( \sigma_2 \sigma_1 \sigma_2 \) each describe a right-handed half-twist of the 3 strands. In the figure, we see that the isotopy relating the geometric braids drawn is given diagrammatically by the move sliding the middle strand from left to right, and the crossing of the outer strands from right to left.

The braid relation plays a role in the mapping class groups of more general surfaces as well: certain mapping classes called *Dehn twists* play the role of generators of the mapping class group for closed, orientable surfaces, and satisfy a braid relation under appropriate intersection conditions. Let \( A \) be a simple closed curve (a smoothly embedded circle) on a surface \( S \). A neighborhood of this curve is an embedded annulus. Choosing orientation preserving coordinates, write a point of this annular neighborhood as \((e^{i\theta}, t)\). Then a right-handed *Dehn twist* \( T_A \) is the isotopy class of the map which is supported on \( A \), and given in these coordinates by

\[
(e^{i\theta}, t) \mapsto (e^{i\theta+2\pi i\varphi(t)}, t), \quad \varphi(t) = e^{\frac{1}{2t}} + e^{\frac{1}{2(1-t)}}.
\]

The effect of a Dehn twist is better understood visually, as in Figures 8 and 9. The former shows the action of the above map on a transverse arc in an annular neighborhood of the circle \( A \). We can also understand the action in another way: imagine cutting the surface along \( A \), twisting one side of the cut surface a full turn, and then re-gluing the surface. If a curve meeting the cut is twisted so as to "turn to its right" as it approaches \( A \), then the Dehn twist is *right-handed* or *positive*; otherwise, it is *left-handed* or *negative*. Negative Dehn twists are the natural inverses of positive twists. Note that a positive arc-half-twist is a square-root\(^3\) of the positive Dehn twist along a circle bounding a disk neighborhood of the arc.

\(^3\)Let \( N \) be a disk neighborhood of the arc, and \( \partial N \) its boundary circle. Then, by replacing \( 2\pi \) in the Dehn twist formula above with \( \pi \), one recovers a formula for the effect of the arc-half-twist on an annular neighborhood of \( \partial N \). This extends to the interior disk by the rotation map \( z \mapsto e^{i\pi}z = -z \). Euler strikes again!
To explore further...

- Emil Artin's foundational work on braids can be found in the article *Theorie der Zöpf*, his 1925 contribution to the fourth volume of *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg*, and in his 1947 *Annals of Mathematics* article *Theory of Braids*. The last is written in English, expanding on and correcting the proofs of his earlier, German-language article. The first treatment to employ the idea of configuration space fundamental groups is found in the article *The braid groups* by R. Fox and L. Neuwirth, which appeared in volume 10 of *Mathematica Scandinavica*. They reproved Artin's presentation rigorously via more modern algebraic topology. For an even more modern and thorough exposition appropriate for algebraically-inclined graduate students, one should consult *Braid groups* by Christian Kassel and Vladimir Turaev. A condensed and geometric approach to braids can be found in Seiichi Kamada's *Braid and knot theory in dimension four*, which generalizes the discussion to braided and knotted surfaces in 4-dimensional spaces.

Braid groups also generalize to surface braid groups, by replacing the disk with other surfaces, such as annuli, spheres, tori, and higher genus surfaces. There is a strong algebraic relation between surface braid groups and mapping class groups of surfaces with punctures, captured first by the Birman exact sequence, and second by Birman-Hilden theory founded in two papers: *On the mapping class groups of closed surfaces as covering spaces* by Joan Birman and Hugh Hilden, which appears in the book *Advances in the theory of Riemann Surfaces* (Proc. Conf., Stony Brook, N.Y., 1969), and their 1973 *Annals of Mathematics* article *On isotopies of homeomorphisms of Riemann surfaces*. By realizing certain surfaces as a double branched covers of punctured surfaces, the theory allows one to study certain highly symmetric mapping classes, called hyperelliptic mapping classes, via surface braid groups. These mapping classes and certain relations among them are frequently used in the study of Lefschetz fibrations on 4-manifolds. Those wishing to study mapping class groups of surfaces in greater detail will find *A Primer on Mapping Class Groups* by Benson Farb and Dan Margalit to be an indispensable resource, including its own concise coverage of the connections between braid groups and mapping class groups.

Certain matrix relations in statistical mechanics known as the Yang-Baxter equations are another instance of a braiding phenomenon. In categorical algebra one may meet the generalized or quantum Yang-Baxter equations, which are built into the definition of braided monoidal categories.
and find use in the study of Hopf algebras, quantum groups and quantum invariants of knots and 3-manifolds. J. Scott Carter and Masahico Saito’s book *Knotted surfaces and their diagrams* gives several combinatorial approaches to knotted and braided surfaces that generalize the theory for classical knots and braids, before expositing on the higher-categorical analogue of the Yang-Baxter equations. Turaev’s magnum opus, *Quantum invariants of knots and 3-manifolds*, takes a multifaceted look at the interplay between braid groups, knots, 3-dimensional topology, quantum groups, category theory and representation theory.

- Cryptographers studying noncommutative alternatives to the popular number-theoretic public-key cryptographic primitives have found possible utility for braid groups and related combinatorial problems in developing new means of securing information. The future of applied braid cryptography is uncertain, as the security of some of the earliest methods has been called into question in light of certain proposed attacks. Yet, there remain candidates which, given an understanding of the space of suitably secure parameters, are potentially suited to low computational resource authentication applications such as RFID verification and security of Internet of Things technology. Of the many papers on the theory of braid-based cryptography, a good place to start is Patrick Dehornoy’s *Braid-based cryptography*, appearing in volume 360 of the American Mathematical Society’s *Contemporary Mathematics* series. The academic arguments about the general security of braid-based methods are still playing out, but one can find some developments in the following articles:


The editors encouraged Andrew Havens to prepare this visually appealing article on braids and mapping classes. The author, whose dissertation research concerns the topology and geometry of 4-dimensional spaces and the ways surfaces can knot smoothly and topologically within them, thanks R. İnanç Baykur for introducing him to mapping class groups, and for subsequently teaching a graduate topics course on the subject last spring.
1 Introduction

This year's Applied Math Master's Project investigated the detection, classification and growth of breast cancer tumors. The project was a collaborative effort by students Connor Amorin, Gabriel P. Andrade, Sandra Castro-Pearson, Abdel Kader Geraldo, Brandon Iles, Dean Katsaros, Terry Mullen, Sam Nguyen, Oliver Spiro, and Melissa Sych under the guidance of Professor Nathaniel Whitaker.

According to statistics from the CDC, breast cancer is the most common cancer diagnosis among women in the United States. Currently, radiology is an expert-based field where automated tools are at best relegated to the role of "second reader." Early detection is an enormously important part of breast cancer treatment, so our goal in this project is to create a machine learning pipeline for detection and diagnosis from mammogram images. We also model the growth and treatment of tumors using a system of ODEs.

Due to a lack of human data, this last part of the pipeline uses data from experiments on lab mice, and is not restricted to breast cancer.

2 Image Dataset

The Curated Breast Imaging Subset of DDSM is an updated and standardized version of the Digital Database for Screening Mammography (DDSM). The database consists of 2,620 scanned film mammography studies. In addition to the mammography images, it contains the verified pathology information associated with each study. The images have been compressed and converted to DICOM format.

The DDSM is one of few well-curated data sets available for research and Computer-Aided Diagnostics (CAD). Although useful, the set has problems concerning data size and noise, as the images are digitized from film. Our work follows a two-fold procedure. First, we implement a three-step image processing method for detecting abnormal masses in mammograms. Once we detect an abnormality, we seek to classify the mass as malignant or benign using classical machine learning techniques along with the histogram of oriented gradient feature extraction method and also using deep neural networks.

3 Detection

Tumors can be recognized as locally low density areas on mammograms, but they may vary in size, shape, image quality and more. Our image detection system consists of three distinct steps. First, we modify the local contrast of each pixel with a linear enhancement filter. Next, we subtract this enhanced image from the original and obtain an image with segmented masses. The third step binarizes the segmented image using a technique called adaptive local thresholding.

In order to enhance our original image, a transformation of pixel values is used which can be described in the following way:

Given a constant \( \alpha \), pixels in the original image \( OI(i,j) \) are transformed as follows to give pixels in the enhanced image \( E(i,j) \):

\[
\text{If } OI(i,j) < \alpha, \quad E(i,j) = a \log[1 + bOI(i,j)] \\
\text{If } OI(i,j) > \alpha, \quad E(i,j) = \frac{1}{b} \left[ \exp\left(\frac{OI(i,j)}{a}\right) - 1 \right]
\]

where \( \alpha \) and \( a \) are chosen empirically, \( b = \frac{1-\exp(m/a)}{m} \), and \( m \) is the maximum grey level. The log function enhances the lower grey level (darker areas), and the inverse function enhances the higher grey levels (brighter areas).
Segmentation of mass regions from normal tissue is achieved by subtracting the enhanced image EI from the original image OI. The segmented image, SI is the result.

Masses are generally more dense than the surrounding tissue, resulting in a bright spot on the segmented image. The image is binarized using adaptive local thresholding. For each pixel in the segmented image SI(i,j), the following criteria is used to classify the pixel as a potential mass pixel or normal pixel by the following rule:

\[
\text{If } SI(i,j) \geq TH(i,j) \text{ and } SI_{\text{diff}} \geq M_{\text{voisi}} \rho, \text{ then } SI(i,j) \text{ belongs to a suspicious area, else it belongs to normal tissue.} \quad TH(i,j) \text{ is the threshold value calculated by:}
\]

\[
TH(i,j) = M_{\text{voisi}} \rho + \gamma SI_{\text{diff}}
\]

where

\[
SI_{\text{diff}} = SI_{\text{max}}(i,j) - SI_{\text{min}}(i,j).
\]

\[
M_{\text{voisi}} \rho \text{ is the average pixel intensity in a small area around the pixel } SI(i,j), \text{ and } SI_{\text{max}} \text{ and } SI_{\text{min}} \text{ are maximum and minimum intensity value in a large window drawn around the area, and } \gamma \text{ is a thresholding coefficient that is chosen empirically.}
\]

4 Classification

Once an abnormality is detected, our aim is to classify it as either malignant or benign. To achieve this goal, we look to two classical machine learning techniques. Here, we use support vector machines and logistic regression with and without principle component analysis (PCA). These models were trained on HOG features as described in Ergin and Kilinc’s A New Feature Extraction Framework Based on Wavelets for Breast Cancer Diagnosis.

A support vector machine (SVM) aims to find the hyperplane which maximizes the distance between the hyperplane and the nearest sample. Effectively, it separates the two classes, and creates the largest possible margin between the edge cases. Logistic regression is a special case of regression with a binary response variable. In logistic regression, the probability or odds of the response taking a particular value is modeled based on the combination of values taken by the predictors.

SVM achieved a 10-fold cross-validation accuracy of 63% for classifying mammogram images as having a benign or malignant tumor. Logistic regression performed poorly, obtaining 54% accuracy. Logistic regression with principle component analysis (PCA) performed the best, with an accuracy of 69%. These accuracies are not satisfactory, especially for medical purposes. We determined that our feature space was limiting our models, and for that reason we looked to neural networks.

To understand the neural network approach to the image classification task, we first need to pose the problem in a well defined manner.

Let \( M \subseteq [0,1]^{w \times h} \) be the space of mammogram images that are at most \( w \) pixels wide and \( h \) pixels high and let the labels \{-1,1\} represent a benign or malignant diagnosis respectively. Suppose there exists an optimal classification function \( f^* : M \rightarrow \{-1,1\} \). Then a neural network ultimately tries to fit an approximation function \( f \) to this true classifier function \( f^* \). To this end, neural networks build \( f \) from the ground up by composing functions known as layers which in turn are formed from simple atomic functions known as units. The units \( g : \mathbb{R}^m \rightarrow \mathbb{R} \) are functions parametrized by a weight vector \( w \) and a scalar bias \( b \). If \( h \) is a fixed nonlinear function \(^4\) called an activation.

\[
g(x; w, b) := h(w^T x + b)
\]

A layer is then an aggregation of units into a column vector. If \( d \) is the dimension of this vector (called the width of the layer), then

\[
f^{(i)} := [g_1^{(i)}, \ldots, g_d^{(i)}]^T
\]

and the network is formed from the composition of these layers, with

\[
f^{(i)} \left( f^{(i-1)} \right) = [g_1^{(i)}(f^{(i-1)}), \ldots, g_d^{(i)}(f^{(i-1)})]^T
\]

Therefore if our network has \( n \) layers

\[
f(x) = f^{(n)}(x) = f^{(n)}(f^{(n-1)}(\ldots(f^{(1)}(x)))))
\]

where \( x \in M \). Once the data has been fed through all of these functions, we normalize the final layer and output \{-1, 1\} based on which class has higher probability. These networks are often represented graphically as in Figure 3. The above discussion gives the neural network prediction function, and it remains to describe the fitting process. We do this via a process known as “supervised learning” where initially we feed the network data \( x \in M \) that is labeled with its true class \( y \in \{-1,1\} \). After the network has been given some of this labeled data we compute a loss function\(^3\) to quantify how poor the network’s predictions are on this data. We then use gradient descent to fit weights and biases to the data so that this loss is minimized.

\(^1\)Also known as neurons.

\(^2\)Usually a rectified linear or sigmoid function.

\(^3\)This is similar to likelihood maximization used in linear regression, etc.
The fully-connected network described above is not the architecture used for state-of-the-art image classification. Instead, we used a type of network known as a **convolutional neural network** (CNN). The functional form differs subtly in CNNs when compared to fully connected networks, but is considerably less intuitive. This is because the inner product in each atomic unit function is replaced by a (discrete) convolution, so

\[ g_{mn}(x; w, b) = h \left( (w \ast x)_{mn} + b \right) \]

\[ (w \ast x)_{mn} = \sum_{k,l} w_{m+k,n+l} \cdot x_{kl} \]

This allows us to retain the 2D structure of each input image. The convolution maps the image to another 2D grid of units (the next layer) as depicted in Figure 4. Convolution is often successful in tasks involving images for numerous reasons such as lowering the number of parameters (which facilitates training), decreasing the representational burden on each unit, taking advantage of the local invariance properties found in most image classification tasks, etc.

Just using a neural network, even a CNN, was not enough for our task. Perhaps one of the biggest drawbacks of neural networks is the sheer volume of data they need to perform acceptably. For this reason researchers have focused much effort into leveraging parameters learned from other classifications tasks with massive amounts of data (e.g. classifying images of cats and dogs). This use of pretrained networks is referred to as **transfer learning**. Essentially, we take a network with already-fit parameters, and then fit some subset of those to our own data.

We used transfer learning on every network we trained for this task, except for a simple baseline CNN that we trained from top to bottom as a point of comparison.

Another method we implemented to help cope with the deficiency of data is known as **data augmentation**. This is the naive, but effective, idea that we can take advantage of symmetries we may see in our data to gain a two pronged advantage: we effectively increase the sample size and we guard against over fitting. Essentially, data augmentation is the creation of data by taking data we have and transforming it in ways that might have appeared if we gather a new data sample. For example, in our domain of mass classification, reflecting a mammogram horizontally is reasonably something we may have seen “in the wild” and so we might use both the reflection and the original to train the network. We augmented with horizontal and vertical flips, as well as small-scale image zooms.

With all of this said, we have the machinery necessary to present our models and results. The first model we implemented and trained on was a simple three-layer convolutional network with two fully-connected layers at the end for classification. We trained this model only on our data as a point of comparison for all of the techniques and massive networks we implemented afterwards. After feeding this baseline the training data 180 times, it achieved ≈ 65% validation accuracy.

Next we used the VGG16 network. This architecture was developed by the Visual Geometry Group at Oxford and performed exceptionally well, given its simplicity, in the famous ImageNet 2014 competition, where it was trained on millions of images curated from the internet. As its name implies, the network consists of 16 layers. After 180 passes of the training data it achieved ≈ 72% validation accuracy.

Finally we used the third version of GoogLeNet. The first iteration of this network won multiple categories of the ImageNet 2014 competition (the same year as VGG16).
It became (in)famous for its clever “inception” modules which act like layers but are each more complicated than our baseline model. This performed with ≈ 78% accuracy after being fed the training data 180 times.

5 Modeling Tumor Growth and Treatment

For this portion of the project we replicated a 2014 paper written by López, Seoane and Sanjuán, called *A Validated Mathematical Model of Tumor Growth Including Tumor-Host Interaction, Cell-Mediated Immune Response and Chemotherapy*. There, the authors consider ODEs from the literature that model interactions among three cell populations: tumor, immune, and host cells. In particular, they non-dimensionalize these equations and focus on tumor-host and tumor-effector interactions since host and immune cell competition is negligible. The paper concludes by incorporating chemotherapy into the model and evaluates the effects on the ODEs using mouse data from Hiramoto and Ghanta (1974).

To understand this system, some intuition of the underlying physical processes is necessary; we begin with the tumor-host interactions. When tumor cells form, they interact with healthy host cell populations and compete for oxygen in the blood stream. Tumor cells multiply at a faster rate, overwhelming and eventually killing the host’s cells. The body recognizes the problem and sends two types of cells to fight off the cancerous ones — natural killers and CD8+ T-lymphocytes — but for small time intervals the two populations can be expressed as a linear combination of one another, and so we refer to them communally as effector cells. Effector cells directly attack the tumor cells and reduce their population. This introduces our second point of comparison: tumor-effector interactions.

Due to discrepancies in units of measure, we non-dimensionalized the ODE. The equations representing the growth rates of the tumor ($\dot{x}$), host cells ($\dot{y}$), and effector cells ($\dot{z}$) then become

$$\dot{x} = x(1 - x) - a_{12}yx - D(x,z)x$$

$$\dot{y} = r_2y(1 - y) - a_{21}xy$$

$$\dot{z} = 1 - d_3z + g \frac{D^2(x,z)x^2}{h + D^2(x,z)x^2} - a_{31}xz$$

where $D(x,z) = d\frac{\lambda x^3}{x^3 + \lambda^3}z^3$.

It is also worth noting that, in the original ODEs, the host and tumor cells are assumed to follow a logistic growth.

Now that we had the ODEs in a form we could work with, we began by trying to understand the qualitative behavior of the system. We plotted nullclines and found the fixed points of this system of equations using given parameter values. We had three saddle points at (0.06, 0.65), (0.1, 0.74, 3.02) and (0, 0, 8.93), and two stable fixed points at (0.65, 0, 0.31) and (0, 1, 8.39). The saddle points are physically uninteresting, but note that the two stable points correspond to total death of the host population and total death of the tumor cell population respectively. After plotting this using the same data as the authors, we could see that all of this matched their results.

With this information gained, our next goal was to evaluate it against experimental results and then incorporate chemotherapy into the model. We used data from Hiramoto and Ghanta (1974) that resulted from experiments where they injected mice with tumor cells and recorded the populations of host, effector, and tumor cells over a period of 36 days. Unfortunately the experimenters could not differentiate between host and effector cells when recording the data points, so these cell counts are combined and reported as “healthy” cells. Furthermore, the body needed 10 days before it noticed the tumor cells and began mounting a resistance, while the tumor cells needed about 15 days to begin growing consistently, so the populations were recorded on day 10 and then every 3 days beginning on the 18th day. After the 21st day, the mice began chemotherapy treatment. For simplicity we did not model the effect of the treatment on healthy host cells and instead focused on the tumor cells. The new growth rate of tumor cells is then

$$\dot{x} = x(1 - x) - a_{12}yx - D(x,z)x - (1 - e^{-pu(t-\tau)})x$$

where $\tau$ is the point in time in which the treatment begins to take affect and $u(\theta) = u_{0e}^{-k_0}$ throttles the degree to which the treatment affects the tumor cell population after $\tau$.

Using this data and least squares fitting we solved for the parameters $g$, $d$, and $s$ above. Plotting our equations using the parameters we found resulted in Figure 5. We also plotted the new equation for the tumor cells after the chemotherapy and Ghanta (1974) resulted from experiments where they matched their results.

6 Conclusion

If the group had more time together, there are a few parts of our pipeline that could be improved. The detection and diagnosis stages would benefit from a newer, fully-digital dataset. These are just now becoming available, and will
likely push the field forward. With larger high-resolution data in hand, we would be able to experiment with different architectures, perhaps eschewing transfer learning entirely. The treatment model would also benefit from newer data to compare with our results. If human data becomes available, this would be ideal. Furthermore, an important aspect of chemotherapy is its negative effect on healthy cells, so incorporating this into our models would be beneficial.

Figure 5: Growth of Cells in a Mouse

Figure 6: Decay of Tumor Cells in Mouse after Chemotherapy

STATISTICAL CONSULTING AND COLLABORATION SERVICES (SCCS)

Statistics is integral to the research work of nearly every scientific discipline. Historically, our department offered statistical consulting services to support other researchers. By 2003, administrative issues resulted in the end to these services. But in 2011, then-Head Michael Lavine revived statistical consulting in the Department as the new SCCS. Being a capable and curious statistician, although lacking external funding, he found many projects – both pro bono and fee-based – through our SCCS website and by word-of-mouth, and executed them either on his own or in collaboration with a small number of graduate students.

By the summer of 2016, Professor Krista Gile joined the leadership of the SCCS in the interest of broadening the participation of students and increasing the volume of projects that could be supported. In the 2016-2017 academic year, Gile also offered a 1-credit graduate course on Statistical Consulting. Since then, Professors Gile and Lavine, along with over two dozen student consultants, have assisted with at least two dozen projects, whose clients include UMass student groups, UMass faculty and graduate students, as well as researchers and professionals outside the university. These diverse clients include researchers interested in reducing the epidemic of prescription opioid use, understanding mathematical patterns in music, and studying the relationship between exercise and academic performance.

By the summer of 2016, Professor Krista Gile joined the leadership of the SCCS in the interest of broadening the participation of students and increasing the volume of projects that could be supported. In the 2016-2017 academic year, Gile also offered a 1-credit graduate course on Statistical Consulting. Since then, Professors Gile and Lavine, along with over two dozen student consultants, have assisted with at least two dozen projects, whose clients include UMass student groups, UMass faculty and graduate students, as well as researchers and professionals outside the university. These diverse clients include researchers interested in reducing the epidemic of prescription opioid use, understanding mathematical patterns in music, and studying the relationship between exercise and academic performance.

In addition to supporting the work of these clients, the SCCS gives graduate (and a few undergraduate) students an opportunity to experience the wide variety of applications of statistics. Seeing real data, while working on a project that a client cares about and communicating with that client, offers our students great practical training for the data science careers of the future.

Unparalled mathematician Maryam Mirzakhani, former Professor at Stanford University, succumbed to breast cancer in July 2017. Mirzakhani was the first woman to receive the Fields Medal in mathematics, earning the prestigious award for her contributions in the fields of Teichmüller dynamics, hyperbolic geometry, and ergodic theory.

Mirzakhani photo credit: Stanford University
GRADUATE PROGRAM HONORS

Three graduate students were honored with the department’s Distinguished Thesis Award this year.

Isabelle Beaudry completed her thesis *Inference from Network Data in Hard-to-Reach Populations* under the supervision of Professor Krista Gile. She studied several significant issues in statistical inference from Respondent-Driven Sampling, a network-based survey sampling technique used to make (approximately) valid statistical inference about hard-to-reach human populations. She is now an Assistant Professor at the Pontificia Universidad Catolica de Chile.

Kostis Gourgoulias wrote his thesis *Information Metrics for Predictive Modeling and Machine Learning* with the guidance of Professors Markos Katsoulakis and Luc Rey-Bellet. He studies the use of information criteria to balance computational and theoretical trade-offs in modeling. Kostis has joined the Data Science/AI team at Babylon Health in London, England.

Zijing Zhang finished her thesis *Statistical Methods on Risk Management of Extreme Events* supervised by Professor Hongkun Zhang. She investigates financial risk using schemes for extreme-value modeling, as well as techniques from copula modeling. Zijing is currently working as a Senior Statistician at the Takeda Development Center Americas in Cambridge, Massachusetts.

The department’s Distinguished Teaching Award this year honors Andrew Havens, in recognition of his outstanding teaching throughout our calculus curriculum, as well as his teaching experience at Amherst College and his contributions to the Math Club.

Andrew Havens, Zijing Zhang, and Konstantinos Gourgoulias with Professors Tom Weston, HongKun Zhang and Markos Katsoulakis
On 12 April 2017, the Department of Mathematics and Statistics celebrated the accomplishments of our top students at our annual Awards Dinner. This evening honors the winners of the Jacob-Cohen-Killam Mathematics Competition and the M.K. Bennett Geometry award, as well as our REU participants, members of the Putnam Competition team, and other students deserving special recognition. Together with the family and friends of the awardees, we were joined by alumni John Baillieul ’67, Roy Perdue ’73, and faculty member emeritus Eleanor Killam.

The evening began with refreshments and dinner. Several problems included on the program inspired animated discussions, both mathematical and terrestrial. The awards portion of the event opened as our guests were tucking in to their apple crisp à la mode, with greetings from our department head Farshid Hajir, and from the MC for the evening, Professor Paul Hacking.

This year the department had a bumper crop of REU (Research Experience for Undergraduates) students, thanks in large part to the continued generous support of Joan Barksdale ’66. Professor Matthew Dobson, the mastermind of our REU program, recognized the following students for their research contributions: Scott Destromp, William Dugan, Jimmy Hwang, Kexin Jin, Kai Nakamura, Kevin Quirion, Cristian Rodriguez, Haley Schilling, and Tran Thu in Pure Math; Anya Conti, Jake Reiser, Ryan Ross, Brandon Whitchurch, and Hui Xu in Applied Math; and Gabriel Clara, Jing Feng, and Mei Jing Lim in Statistics.

The M.K. Bennett Geometry Award is presented to the student who exhibits the best performance in our Geometry course Math 461. This award honors the memory of Professor Mary Katherine Bennett. Professor Bennett earned the first Ph.D. in our department in 1966. After teaching at Dartmouth College, she returned to UMass Amherst for the rest of her career where she encouraged interest in geometry and high school teaching among undergraduates. The course she developed now covers Euclidean, spherical, and hyperbolic geometry, and is taken by all our math majors in the teaching track. Professor Tom Braden presented the winners Shelby Cox and Jared Yeager with their awards.

Shelby Cox, Aaron Dunbrack, and Alexander Fischer were recognized for competing in the 2016 Putnam Exam. The team placed 55th out of 568 institutions. Aaron's performance was truly outstanding, placing 113th out of 4164 contestants. He will begin a PhD in Physics at Stony Brook this Fall, home of the prestigious Simons Center for Geometry and Physics. (Shelby is a Junior and Alex is a freshman, so we look forward to more stellar Putnam performances from them!)
The Jacob-Cohen-Killam Mathematics Competition is named in honor of the memories of Professors Henry Jacob and Haskell Cohen, and of the continuing contributions of Professor Emeritus Eleanor Killam. These three faculty members have kindled interest in mathematics among undergraduates through annual mathematics contests.

The competition is open to first and second year students. Each year around twenty contestants attempt to solve ten challenge problems conjured up by our faculty members. This year the competition was again generously sponsored by John Baillieul ’67, Roy Perdue ’73 and James Francis ’86.

Professor Franz Pedit awarded this year’s first prize of $1600 to James Hagborg; the second prize of $1000 to Alexander Fischer, the third prize of $600 to Artem Vysogorets, and the fourth prize of $200 to Daniel Weber. Franz explained that the problems, while only requiring a minimum of mathematical knowledge, force the students to work creatively and connect with important research topics in higher mathematics.

Professor Farshid Hajir presented the Fraser Community Scholarship to Julianne Rose Higgins, and the Leon Emory Lincoln and Robert Bradley Lincoln Scholarship to Shelby Cox.

In addition, Professors Anna Liu, Siman Wong, and Farshid Hajir recognized the Outstanding Academic Achievement of Juniors Athena Higgins and Shelby Cox; Seniors Sam Castillo, Erica Doyle, William Dugan, and Cristian Rodriguez; and Math teaching majors Julie Arakelian and Andrew Marton. Professor Hong Kun Zhang presented the Don Catlin Award for Outstanding Achievement in Applied & Computational Mathematics to Radha Dutta and Hujing Yu.

Graduate program director Professor Tom Weston recognized the winners of the Graduate Student Awards: Isabelle Beaudry, Kostis Gourgoulias, and Zijing Zhang won the Distinguished Thesis Award, while Andrew Havens received the Distinguished Teaching Award.

We are fortunate to have several awards made possible by generous donations!
The evening ended with closing remarks by Steve Goodwin, Dean of the College of Natural Sciences. He spoke warmly of the successful evening and of the great strides the department has made in recent years under the tutelage of head Farshid Hajir, including doubling the number of math majors! This will be Steve's last year as Dean before returning to the Department of Microbiology, and Farshid thanked him on behalf of the whole department for his inspiring leadership over the last 8 years.

The program for the dinner included a problem from the Putnam exam and two from the Jacob-Cohen-Killian competition.

**Putnam exam:** Consider a \((2m-1) \times (2n-1)\) rectangular region, where \(m\) and \(n\) are integers such that \(m, n \geq 4\). This region is to be tiled using tiles of the two types shown:

![Tiles](Image)

(The lines divide the tiles into \(1 \times 1\) squares.) The tiles may be rotated and reflected, as long as their sides are parallel to the sides of the rectangular region. They must all fit within the region, and they must cover it completely without overlapping.

What is the minimum number of tiles required to tile the region?

(Friendly hint: To get started on this challenging problem, first try the case \(m=n=4\).)

**JCK competition Problem 1:** Calculate the probability that a randomly drawn triangle is acute (all angles less than 90 degrees).

**JCK competition Problem 2:** Pick four “generic” vectors \(a, b, c, d\) in the plane such that \(a + b + c + d = 0\). Concatenating them in all possible orders defines 24 quadrilaterals in the plane, which fall into three types: some are convex; some are nonconvex but not self-intersecting; and some self-intersect like a bow-tie. What fraction of each kind will occur?

Distinguished Teaching Award winner Andrew Havens with Math Director of Administration and Staff Ilona Trousdale

Christian Rodriguez and William Dugan each completed an Honors thesis
Solutions to Last Year’s Challenge Problems

Now the summer is gone,
As if it has not been here.
In the sun it is warm.
But it is not enough.

Everything that could come true
Like a five-fingered leaf
Fell straight into my palm,
But it is not enough.

Neither evil, nor good
Was lost in vain,
Everything burned in light,
But it is not enough.

Life kept me under its wing,
Took care of me and saved me,
I was lucky indeed.

But it is not enough.
The leaves were not burnt,
The branches were not broken...
The day is clean like glass.

In Andrei Tarkowski’s film Stalker this poem by his father Arseny is recited by the Stalker to his two companions, the Physicist and the Poet, during their expedition into the Zone, a landscape in which common laws of reality are mysteriously altered. Without even trying to provide an exegesis of the film – or the poem – their imagery and atmosphere evokes an affinity with mathematics, embracing liberal arts and natural sciences, mind and body, thought-reality and material-reality, creativity and discovery. To imbue the next generation with a sense of wonder and awe about the unique position mathematics holds in human endeavors, to make them aware of the complexities, unexpectedness, and beauty of the mathematical landscape, and point them to a realm which lies at the foundations of creation, may be the most important contribution in our educational process. It does not come as a surprise that the laws of our perceived reality are written in terms of mathematical structures. To understand the underlying structure of Nature requires the most refined – the most abstract, the hardest-to-discover – mathematical concepts. Or in the graffiti version: physicists want to be god, and god is a mathematician.

Can such a “puristic” view of mathematics be maintained in an educational system and society that leans more and more towards uniformization, regularization, standardization, and top-to-bottom procedures? Of course, one can always take solace in Miguel de Cervantes’s Don Quixote: while designing the Jacob-Cohen-Killam exam (whence the challenge problems are drawn) and writing this column, your Problem Master occasionally feels the breeze of windmills.... It’s no accident that most of these problems are slightly-ambiguously phrased. To find a satisfactory interpretation of the question is often part of the problem. Solutions rarely require any technical knowledge, memorized formulas and routine algorithms are of little help. What is needed is a creative “subversiveness” and an inexplicable urge to “take the other road.” As Reinhold Messner (echoing George Mallory) once said when asked why he climbs mountains: “because they are there.”

And so we now turn from poetic reflection to the challenge problems themselves:

Problem 1. Which of the numbers $\sqrt{2016}$ and $\sqrt{2015}$ is the larger one?

Comparing the sizes of two numbers starts at an early age: is it a better deal to share 12 slices of pizza among 7 children or 16 slices of pizza among 11 children? And the children quickly get the idea that comparing two numbers in size amounts to deciding whether their ratio is smaller or greater than one. Putting $a = \sqrt{2015}$ and $b = \sqrt{2016}$ we need to compare $a^b$ with $b^a$. In the 16th century the Scottish mathematician, physicist and astronomer John Napier of Merchiston (1550–1617) developed the idea of logarithms to simplify calculations with products and powers. The (natural) logarithm of a positive number $x$, denoted by $\log x$, is the power by which one has to raise Euler’s number $e = 2.71828...$ to obtain $x$. In formulaic language this reads

$$x = e^{\log x}$$

or, equivalently

$$\log e^x = x$$

for $x > 0$. A direct, unwieldy computation using arithmetic properties of log can solve this challenge problem; instead, follow the elegant path provided by Mark Leeper, BA ’72. Raising $a^b$ and $b^a$ to the power $\frac{1}{ab}$ which preserves the ordering, we need to compare $a^{\frac{1}{ab}}$ with $b^{\frac{1}{ab}}$. This amounts to whether the function $f(x) = x^{\frac{1}{ab}}$ is increasing or decreasing for $x$ in an interval containing both $a$ and $b$. Thus we must determine the sign of the derivative $f'(x)$. Since the logarithm is an increasing function, we may as well calculate the sign of the derivative...
of the function \( \log f(x) = 1/x \log x \). Now

\[
(\log f)'(x) = \frac{1}{x^2} (1 - \log x)
\]

which is negative as long as \( x > e \). In particular, we deduce \( 1/a \log a > 1/b \log b \) and therefore \( a^{1/e} > b^{1/e} \) for \( e < a < b \), which implies \( \sqrt[2015]{\sqrt[2016]{\sqrt[2017]{\cdots}}} > \sqrt[2016]{\sqrt[2017]{\cdots}} \).

Mark also offered a pertinent personal remark: “ […] this problem hits close to a problem I set for myself when I was taking calculus in high school. I saw that \( 2^4 = 4^2 \). I said that \( x \) is an exponent image of \( y \) if \( x^y = y^x \) for \( x \neq y \). Every number greater than 1 has an exponent image except for Euler’s number \( e \).” The reader is invited to think about this characterization of Euler’s number.

**Problem 2.** 9 points are given inside a square of side length 1. Prove that 3 of the points determine a triangle of area at most 1/8.

One way to quickly see this is by dividing the square in four squares of area 1/4 (like in a window pane). Then there have to be at least 3 points in at least one of the four small squares. Form a triangle with these 3 points and observe that the area of triangle in a square is at most half of the area of the square.

**Problem 3.** Imagine three equally-charged particles constrained to lie on a circular wire loop which repel each other with a force which decreases monotonically with their distance (in the plane of the circle). Determine the equilibrium configuration – meaning the total force on each particle exerted by other particles is normal to the circle – and show that there is only one such configuration.

For a configuration to be in equilibrium, that is, all of the particles are at rest, by Newton’s law of motion the resulting force on each particle has to have no tangential (to the circle) component. An equilateral triangle configuration achieves that, but the question is whether there is another equilibrium configuration. Drawing the resultant force two of the particles \( P_1 \) and \( P_2 \) exert on a chosen third particle \( P_0 \), one sees that the two particles have to be equidistant (measured in radians) from \( P_0 \), an isosceles triangle configuration. Applying the same reasoning to the particle \( P_1 \) shows that the three particles have to be equidistant in order to be in equilibrium. (For those who like calculations, assume the circle has radius 1 and use complex numbers – as well as rotational invariance of the problem – to parameterize the positions of the particles by \( P_0 = 1, P_1 = e^{i\alpha}, P_2 = e^{i\beta} \). Then, under the assumption that the repelling forces between the particles decay monotonically with distance in the plane, one obtains \( \alpha = \beta \).

Incidentally, the motion of 3 (or any \( n \geq 2 \)) particles on a circle, now connected by “springs” whose spring constants decrease exponentially with distance between the particles, is one of the fundamental completely integrable equations of theoretical physics, the (periodic) Toda equations. Originally developed by the Japanese physicist Morikazu Toda (1917–2010) as a simple model for a one-dimensional crystal in solid state physics, those equations are also intrinsically related to Lie algebras, Lie groups and their representation theory: the original open Toda equations relate to classical finite dimensional Lie algebras; and the periodic Toda equations, to the infinite dimensional loop (affine) Lie algebras. Our case of 3 particles on a circle corresponds to the loop Lie algebra of trace-free \( 3 \times 3 \) matrices. This case has been pivotal in classification problems of surfaces in 3-space, such as minimal surfaces, constant curvature surfaces, Willmore surfaces, and affine spheres, to name a few. Besides its impact on “pure” mathematics, there are deep connections of the Toda model with problems in soliton theory, statistical physics, quantum physics and string theory that almost surely were not anticipated by its founder. This provides yet another example of some of the conceits in the introduction of this “solution sheet,” namely the universal, overarching fabric that abstract mathematical concepts offer.

**Problem 4.** Cosmic rays strike Earth often, some tangentially (angle zero degrees), some perpendicularly (angle 90 degrees = \( \pi/2 \)), and some at angles in between. Assuming 1) Earth is perfectly spherical, and 2) rays are equally likely to come from any direction, what is the most likely strike angle?

Or equivalently (math version):

What is the most likely angle that a random line in 3-space meets a round 2-sphere?

When trying to parse the non-mathematical formulation of this problem one could get confused: if rays come equally likely from all directions “what is the most likely strike angle” sounds almost like an oxymoron. As one of our readers, Doug Bosworth, MS’88, whose solution we follow closely, remarked...
about “random” moves in chess: one can choose randomly from a list of legal moves, or one can randomly choose a piece to move, and then randomly pick a legal move for that piece.

To unravel this confusion for our problem, the mathematical formulation is helpful: “rays coming equally likely from all directions” implies that we can calculate the “amount” of rays striking the earth – which we model by a sphere of radius 1 – by first calculating the “amount” of rays hitting the earth in a fixed direction, and then multiply this number by the “amount” of directions 4π. In more mathematical terminology, we equip the space of rays striking the earth with a measure invariant under the 3-dimensional group of rotations in space.

The “amount” of rays hitting the unit sphere in a fixed direction (say vertical) is the area π of the disk of radius 1, the projection of the unit sphere to the horizontal plane. Therefore the “amount” or – mathematically speaking – the volume of the space of rays hitting the unit sphere is 4π. Now picture the sphere in your mind’s eye with that vertical column of rays hitting the upper hemisphere. The strike angle at a location on the upper hemisphere is the angle between the vertical and the tangent plane to the sphere at that location. Thus the “amount” of vertical rays striking at angles σ between 0 (the equator) and some latitude θ ≤ π/2 (Amherst has θ ≈ 42°) is the area π(1 − cos²θ) = π sin²θ of the annulus of inner radius cos θ and outer radius 1.

Whence
\[
\frac{\text{area(annulus)}}{\text{area(disk)}} = \sin^2 \theta = \int_0^\theta S(\sigma) d\sigma
\]

is the probability that the strike angle σ is between 0 and θ. By the fundamental theorem of calculus, the probability distribution S(σ) – in other words the likelihood that the rays strike at angle σ – is the derivative

\[
S(\sigma) = \frac{d}{d\theta} \int_0^\theta S(\sigma) d\sigma = \frac{d}{d\theta} \int_0^\sigma \sin^2 \theta = 2 \sin \sigma \cos \sigma = \sin(2\sigma)
\]

which attains its maximum at σ = π/4. Therefore the most likely strike angle is π/4 = 45°.

(This result is sometimes called Shoemaker’s Theorem, named for the planetary scientist after whom a well-known comet is also named. The same result was discovered by the geoscientist G.K. Gilbert about 70 years earlier.)

Problem 5. Three velociraptors are at vertices of an equilateral triangle and you are at the mid-point of the triangle. One of the raptors is injured and his speed of pursuit is 1/3 that of the healthy raptors’ speed, which is 4 times your running speed. Which way should you run to stay alive the longest? Velociraptors pursue in the direction of the line of sight to the target.

Among the six problems, this is perhaps the most fun, the most captivating, and the one that can cause you sleepless nights (the students taking the JCK exam don’t have that luxury). The problem is taken from the xkcd comic strip The Substitute by Randall Munroe, who also wrote the book What if? The problem may not directly point towards deeper conceptual connections in mathematics, but it does help you to survive a few seconds longer if you get it right. Were you in the situation of the problem, you may not be able to keep your cool and think through it, but we already mentioned Cervantes.... This problem is very ambiguously stated and needs some re-interpretation for students to even have a chance to solve it in a reasonable amount of time. Let us discuss some (and by no means all) of the issues with the problem: there is no mention of the size of the triangle (which may not really matter); there is no information about whether the velociraptors and the human accelerate up to, or immediately run at, their maximal speeds (which may be negligible for the answer); there is no information about which trajectory the human is taking (straight line, zig-zags, some random path...). The question “which way should the human run” could be taken literally and then it begins to sound like a statistical mechanics problem. A simplifying assumption is that the human runs along a straight line with constant velocity from the start. Assuming also that the velociraptors pursue with their maximal speed from the start “in the direction of the line of sight” their trajectories can be calculated by a differential equation which has the tractrix curve as a solution. It gives the shortest capture time among all possible paths the pursuer can choose. This can be done with basic knowledge of calculus and differential equations,
but one has to be really good at it in order to get the answer in the allotted 2 hours for the JCK exam.

Having said all this, we propose a simplified model in which the raptors see into the future and know along which straight line they have to run in order to intercept the human. Let $L$ denote the distance between the human and the raptors and $v$ the velocity (in units of $L$ per second) of the human. Then $v_1 = 4v$ is the healthy raptors’ speed and $v_0 = 4/3 \, v$ is the injured raptor’s speed. With the situation as in the picture, the intercept conditions of the trajectories are given by

$$t v e^{i \theta} = L + t v_0 e^{i \theta_0},$$

and, by symmetry

$$t v e^{i \theta} = L + t v_0 e^{i \theta_0 - 2/3 \pi}.$$

Here $\theta$ is the angle of the human’s path and $\theta_0, \theta_1$, the angles of the raptor’s paths, measured from the horizontal axis. Due to the symmetry of the situation, we only have to consider two raptors, $R_0$ (injured) and $R_1$ (healthy), so that $0 \leq \theta \leq \pi$. From the above equations one can solve for the intercept times $t_0(\theta) = \frac{L}{v \cos \theta + \sqrt{v_0^2 - v^2 \sin^2(\theta)}}$ and $t_1(\theta) = \frac{L}{v \cos(\theta - 2/3 \pi) + \sqrt{v_0^2 - v^2 \sin^2(\theta - 2/3 \pi)}}$.

The interested reader may want to verify that this conclusion pertains for the more realistic tractrix (curve of pursuit) trajectories, the only difference being that you stay alive slightly shorter. So knowing the future for the raptors is not really an advantage, since they have to wait a bit longer to get their meal. The ambitious reader should next allow acceleration (perhaps the same conclusion will hold?) and the really ambitious reader may want to allow the human to randomly change directions (it may be easier to make a turn when you move slowly).

### Problem 6

**Problem 6.** Prove that there does not exist a convex polyhedron such that all of its faces are hexagons. (A polyhedron is a solid in 3 dimensions whose boundary consists of a union of polygons. We say a polyhedron is convex if for any two points of the polyhedron the line segment joining the points is contained in the polyhedron. A face of a convex polyhedron is a polygon contained in the boundary which is equal to the intersection of the polyhedron with a plane.)

We tacitly understand (or make the additional assumption) that the angle between two edges at a vertex of a polygon is not equal to $\pi$, that is, a triangle with 3 points added on each of its sides is not considered a hexagon. Otherwise one can construct obvious counterexamples to the statement of the problem. In addition, a vertex (corner) of the polyhedron is always considered to be a vertex of any of its incident polygons. One could also make the much more restrictive assumption that all hexagons are regular (and congruent). In that case Mark Leeper, who provided the solution “from the book” – as Erdős would have said – to the first challenge problem, gave also a nice argument for this problem: since there is a hexagonal tiling of the plane (maybe you see this in your bathroom) at every vertex of the polyhedra the incident hexagons would be in the same plane. This could be an entry point to the classification of Platonic solids, that is,
polyhedra whose faces are congruent regular polygons, or the tiling problem of the Euclidean plane (a trip to Alhambra in Granada, Spain, is highly recommended) or, more generally, the tilings of the round sphere and the hyperbolic plane. (Maybe we postpone this for future challenge problems.)

Coming back to our original problem, you may want to carry out the following experiment in case you have never done so: take a cube and calculate the alternating sum

\[ \# \text{ vertices} - \# \text{ edges} + \# \text{ faces} = v - e + f = ? \]

You will get 2. Now do the same for a tetrahedron, a dodecahedron, or for that matter any polyhedron: no matter what kind of polygons form the faces, you always will get 2. Perhaps you thought of a more interesting polyhedron obtained by drilling a hole through a cube (like coring an apple). Yes, you no longer get 2 . . . . Such child’s play led the Swiss mathematician Leonard Euler (1707–1783) to the fundamental observation that

\[ v - e + f = 2(1 - g) , \]

where \( g \geq 0 \) denotes the number of holes – also called the genus – of the polyhedron. This formula is just the tip of an iceberg of an underlying structure, homology theory, with vast ramifications in all areas of mathematics and physics: topology (Poincare conjecture), algebraic and differential geometry (Riemann-Roch type theorems), Analysis (Hodge theory, Atiyah-Patodi-Singer index theorems), quantum field theories, and so forth.

Inspired by those lofty thoughts we come back to the problem at hand: the number of edges of our polyhedron made out of 6-gons is \( e = \frac{6f}{2} \), where the half comes from the fact that each edge is shared by exactly two faces. Applying Euler’s formula, we thus calculate the number of vertices to be

\[ v = e - f + 2 = 2f + 2 . \]

Another way to compute the number of vertices is the following: at each vertex \( v_i \) let \( b_i \) be the number of faces incident to that vertex. We note that under our assumptions \( b_i \geq 3 \) so that

\[ v = 6f - \sum_{i=1}^{2f+2} (b_i - 1) \leq 6f - 2(2f + 2) = 2f - 2 \]

which is a contradiction. Thus, there cannot be such a polyhedron with all of its faces 6-gons.

We always like hearing from you. While the Problem Master is on sabbatical in Berlin, please send any solutions, comments, or other feedback via email to Professor Franz Pedit <pedit@math.umass.edu> with the subject line “Challenge Problems 2017”. Remember to include your full name, information about your UMass Amherst degree(s) earned, and any other interesting things you wish to share with us.
AVA MAURO REVIEWS NEW BOOK BY ALUMNUS PETER J. COSTA

Applied Mathematics for the Analysis of Biomedical Data: Models, Methods, and MATLAB® by Peter J. Costa, PhD’84, provides a hands-on presentation of data analysis methods applied to biological systems. The book can serve as a textbook for students or a reference guide for industry scientists. The text begins with an overview of methods for processing and analyzing data, including techniques such as data visualization. Subsequent chapters delve into mathematical epidemiology (specifically, modeling the spread of contagious disease), statistical pattern recognition and classification, hypothesis testing in biostatistics, clustered data, and analysis of variance. To facilitate the use of the presented methods in practice, the text contains sample code blocks and explanations of MATLAB functions, and the companion website includes data files and MATLAB script files.

Dr. Costa is Senior Applied Mathematician at Hologic Incorporated in Marlborough, MA, and is the co-creator of MATLAB’s Symbolic Math Toolbox. He has developed mathematical models for the spread of HIV, the outbreak of AIDS, the transmission of an infectious respiratory disease throughout a population, and the diagnosis of cervical cancer. His research interests include scientific computing and mathematical biology. Peter and his wife Anne have been generous donors to the department, sponsoring the Distinguished Lecture in Applied Mathematics in honor of the late Professor Melvyn S. Berger.

NEW FACES IN THE DEPARTMENT

Sohrab Shahshahani joined the Department as an assistant professor in September 2016. Sohrab’s research is in partial differential equations. More specifically, he has worked on nonlinear wave equations which arise in various geometric and physical settings; the tools of his research come from analysis and geometry. One equation which has been a focus of Sohrab’s research is the wave map equation. From a geometric viewpoint this is the Lorentzian analogue of the harmonic map equation which is commonly studied in differential geometry. It can be thought of as an analogue of the usual wave equation where the wave takes values in a curved subspace rather than a linear space. In physics, wave maps arise in a number of different contexts, including in general relativity and in nonlinear sigma-models in particle physics. More recently Sohrab has also been working on the Euler equation for incompressible, inviscid fluid flow. His interest in this direction lies in understanding the motion of gravitating fluid bodies, with free boundaries which evolve with the fluid flow.

Sohrab received his PhD from École Polytechnique Fédérale de Lausanne, in Switzerland, under the supervision of Professor Joachim Krieger. Before coming to Amherst, he spent three years as a postdoc at the University of Michigan, Ann Arbor.

Christine Curtis joined the department as Business Manager in February 2017. Prior to coming to the University of Massachusetts, Christine worked for the FDIC, for the Department of Defense, and as a Controller in the private sector.

Christine grew up in the Cleveland and Cincinnati areas of Ohio before her family relocated to Rochester, New York, in her senior year of high school. She considers Rochester her second home, as she has family who reside there, and she misses her favorite grocery store – Wegman’s – which is headquartered there.

When she is not working, Christine enjoys walking, hiking, kayaking, traveling, antiquing, gardening, long weekends on Lake Winnipesaukee, and spending time with family. She is an avid sports fan: her favorite teams are the New England Patriots and Boston Red Sox.

On her bucket list are trips to Dubai, Australia and New Zealand; VIP passes for a skybox or 50-yard-line seats at Gillette Stadium in Foxboro for a Patriots game; and a meet and greet with Tom Brady.

She resides in South Hadley with her husband and twin sons.
interested in the interdisciplinary aspects of mathematics. Before coming to Amherst, I was trained as an analyst, but I always yearned to indulge my fascination with the applications of mathematics to science. So I focused my research on fluid dynamics, mainly nonlinear waves and vortices, using both analytical and numerical methods.

My main collaborations in the first few years at UMass were with Alex Eydeland and Joel Spruck, both of whom left the Department in the 1990’s for distinguished careers elsewhere. Alex brought the numerical ideas to our work, and collaborating with him was a delight in every way. In one of our bigger projects, together with Joel and a short-term visitor, Alexander Lipton*, we tackled magnetohydrodynamics and plasma physics, subjects that mathematicians tend to shy away from. My PhD students, Sheng Wang and Brian Morris, wrote theses on these topics. It is fair to say that I found my intellectual bearing during those early years in the environment created by colleagues in the Center.

But after some years I became frustrated with the scope of my work in fluid mechanics. While there is great charm in watching real flows in nature, whether ocean waves, vortices in a river, or weather systems in the atmosphere, there is a huge gap between those real flows and what can be handled mathematically. Analysts investigate the abstract properties of the governing equations, while engineers try to solve the equations by brute force on the biggest computers they can get. I was not satisfied with either of those extremes.

Fortunately, I was rescued by serendipity. In the early 1990s, Nate Whitaker and I attended a conference in Princeton at which a speaker from France presented a statistical theory of coherent states in two-dimensional turbulence. We quickly realized that we had a method to compute the statistical states he proposed, and in fact, we worked out the essence of the algorithm while we drove home from New Jersey. Thus began my interest in the statistical theories of turbulence, which have occupied me ever since.

To work in this subject required another period of self-education, as it relied on probability theory and statistical physics. This was exactly the change of perspective that I needed, and so I was happy to take up this challenge. Richard Jordan, Joe Heisler and Zhi Liang did their doctoral work with me at this formative time. Later I enlisted the help of Richard Ellis, whose expertise in large deviation theory allowed us to bring the theory to a polished form. Our many joint papers, some together with Richard’s graduate students Kyle Haven, Christopher Boucher, and Marius Costeniuc, make up the most sustained collaboration of my career. On the applied side, I worked with Andy Majda of the Courant Institute to put the theory into practice for geophysical flows, including a prediction that the embedded Great Red Spot and the zonal jets on the gas giant Jupiter are the most probable, stable states of its turbulent weather layer.

Then in 2002, upon the retirement of the Department Head, Don St Mary, and 13 other senior faculty members, I became the new Head. Even though leading such a big department is an exhausting job, I have always enjoyed variety in my activities, and being Head was certainly a major change of pace for the next three years!

Upon finishing my term, I took a delayed sabbatical and renewed my teaching and research, turning toward nonequilibrium statistical theory, a subject in which I had dabbled for a few years before. I had an embryonic idea for a new approach. Two PhD students, Adam Eisner and Raja Nagaswaran helped me probe into some of its possibilities, and after what seemed a very long time, I eventually brought this idea to fruition. The resulting approach has been the focus of my attention in recent years.

In a nutshell, the problem is to employ statistical mechanics to understand the behavior of dynamical systems that are high dimensional and chaotic, even turbulent. Equilibrium theory discards all the details of the underlying dynamics – except its invariants – and makes statistical predictions from the invariant measure. A nonequilibrium theory attempts to include non-invariant observables in a coarse-grained description, and to predict the average behavior of those coarse observables. This general problem is a central one in mathematical science, and nowadays the applied mathematics community is much concerned with multi-scale problems, dimension reduction and predictive modeling.

But when one turns to the physics literature for help on nonequilibrium matters, one gets frustrated, since no systematic approach comparable to the equilibrium theory exists. Independent minded to a fault, I set out on my own, relying on the general principles of dynamical systems, statistical mechanics and information theory.
Once I had a formulation and some preliminary test results, I was fortunate to receive funding from the National Science Foundation for a post-doctoral researcher to help develop my concept into an actual method. I hired Simon Thalabard, a talented physicist and fluid dynamicist from France on whose Ph.D. committee I sat, and we recently completed several papers that apply my approach to some turbulence models; part of that work was joint with Qian-Yong Chen. It has been most gratifying for me to take part in this rewarding collaboration just before my retirement.

That finally brings me to the present, and to the still unanswered question of how I will spend my time in retirement. For the foreseeable future I plan to continue my current research. Not only do I have ongoing projects with Simon, but I am advising Jonathan Maack on his Ph.D. dissertation, which applies the nonequilibrium method to ensembles of interacting vortices. Longer term is much harder to predict — this is a reality for our lives, as it is for our mathematical models! I am attracted to those problems in the ocean and atmosphere sciences which have motivated my work for two decades. To date most of my results have pertained only to prototypes, not the more realistic models that I would like to be able to treat. There is no lack of challenges there! To meet them I may once again find myself learning some more mathematical physics, which is something that I find enjoyable in itself.

It may seem strange that I have not said anything about my teaching experiences. But this is consistent with the editors’ request that I emphasize the future, which for me is post-teaching. Nonetheless, let me say that it would be impossible to imagine an academic career without all the facets of teaching intertwined in it. For me the academic life is simply continual learning and understanding, or at least trying to do so; and bringing this attitude into the classroom is what constitutes good teaching.

My most satisfying experience in this respect has probably been directing the Masters Program in Applied Mathematics and supervising its annual group projects, which I did for many years. Tackling a new topic inspired from real problems in science, industry or business with a new group of graduate students each year was always thrilling — and often intimidating. It is too radical to extrapolate that approach to a general viewpoint on classroom pedagogy, but I believe that even standard lecture classes benefit from combining theory with practice, and individual with collaborative learning.

As I look forward to my life without the responsibilities of teaching and committee work, I will have more time to indulge my hobbies. First and foremost, music will keep me active and creative. My wife, Nina, a pianist, and I have longtime friends with whom we play music, and to keep up I will need to practice my cello more religiously. And I would like to learn more about music composition, and maybe even try my hand at it, as I did almost 50 years ago. Very recently I have also taken up drawing and sketching. I tend to be a visual person — my mathematical intuition is geometric — and my father was a self-trained artist. At the risk of sounding like a philosopher, I believe that blending the arts, or aesthetic perception, with the sciences, or intellectual understanding, is an enduring aspiration for us humans, no matter how modest our achievements may be. Besides all of that — which might feel daunting at times — it will be fun to travel on a freer schedule, something that both Nina and I particularly enjoy. Throw in an increasing sequence of grandchildren, a total four at the moment, I doubt that I will be sleeping in late!

* Lipton was known as Alexander ”Sasha” Lifschitz at UMass.
ALUMNA PROFILE:
HEATHER HARRINGTON ’06

After graduating from UMass Amherst in 2006, Heather Harrington earned a PhD at Imperial College London and has embarked on a successful career focusing on research in mathematical biology. This is a real success story that the Newsletter is happy to celebrate.

Heather had planned to attend medical school upon graduation, however, she was inspired to attend graduate school in mathematical biology as a result of her extremely positive experiences as a math major in our department: small classes, the interest taken in her career by a number of professors, being an Undergraduate Teaching Assistant from her freshman year, and the extensive support that the department provided. Not only was she encouraged by faculty members to participate in summer programs, but also she was awarded the Goldwater Scholarship, for which the department nominated her.

Most significantly for her future career, Heather was invited to participate in an REU (Research Experience for Undergraduates) project with Professors Panos Kevrekidis and Nate Whitaker on a hybrid PDE model of tumor-induced angiogenesis. One of the themes in her future research in mathematical biology started during that REU project: how to analyze a model whose dynamics are governed by unknown parameter values that are challenging to estimate with limited data. As a result of this project and helpful conversations with Panos and Nate, Heather decided to apply to graduate school in mathematical biology instead of medical school.

In 2006, Heather went to Imperial College London to pursue a PhD. There she was advised by Professor Jaroslav Stark and mentored by Professor Dorothy Buck, thanks to the Enhancing Diversity in Graduate Education (EDGE) summer program to prepare women for graduate school. Nate Whitaker advised Heather to apply for this program, and they paired Heather up with Dorothy, who was on her dissertation committee and later became her second advisor. Heather was a member of the bio-math group and participated in the Centre for Interdisciplinary and Systems Biology, an activity that enabled her to conduct research on biological problems using mathematics and working with data. Heather was granted a partial scholarship from Imperial and was also awarded a NSF Graduate Research Fellowship to support her studies in the UK.

Heather’s PhD dissertation focused on innate immune response and cell death, using dynamical systems approaches with data. She had an opportunity to participate in the Mathematical Biosciences Institute at Ohio State University. There she attended lectures by Jim Keener, worked on a project involving cell death, and met Ken Ho, who was starting as a graduate student at the Courant of Mathematical Sciences at New York University. Using the NSF funding, Heather visited Ken, and they collaborated on two papers modelling cell death. When they proposed a new model based on recent structural data, one of the questions posed by a reviewer was how to discriminate between competing models; a key issue was that even if more data becomes available, it would not be possible to estimate all the parameters. This question motivated Heather and Ken to build upon ideas rooted in chemical-reaction network theory and computational algebraic geometry concerning how to compare models and data without estimating parameters.

Two significant events occurred during Heather’s Ph.D. studies in mathematics at Imperial College London. First, she met her future husband, Mariano Beguerisse Diaz, who was enrolled in the same PhD program. Second, Heather’s Ph.D. advisor, Jaroslav Stark, whom she greatly respected, was diagnosed with terminal cancer. To get through this period of uncertainty surrounding Jaroslav’s illness, she benefited from valuable advice given by her UMass advisors, Nate Whitaker and Panos Kevrekidis, from research projects with Ken, and from support by Mariano. Dorothy
Buck became her second advisor during her final year, and Heather successfully defended her PhD in 2010.

As a result of a serendipitous discussion with Professor Michael Stumpf at Imperial, Heather learned that he was looking for a postdoc in molecular biology to be part of his theoretical-systems biology group. Michael offered members of his group independence to develop new ideas and research directions. Heather joined Michael's group, and he exposed her to additional experimental data and made other collaboration opportunities possible. Simultaneously, Michael taught her Bayesian statistics, which is an important tool for inferring parameters and performing model selection. During that time, Heather also participated in a workshop at the American Institute of Mathematics (AIM) in 2013, where she initiated a number of projects. Through EDGE funding, she had the opportunity to visit Stas Shvartsman and Yannis Kevrekidis at Princeton University.

Heather's postdoc at Imperial was a time of tremendous growth in terms of developing her own research program, with biological questions driving the creation of new mathematical methods. Based on these new research directions, she wrote a number of job applications and applied for several research fellowships. She was selected for a Hooke Fellowship at Oxford’s Mathematical Institute as well as a postdoctoral research fellowship from the UK Engineering and Physical Sciences Research Council (EPSRC, similar to the NSF in the US). Together, these fellowships combined to provide six years of funding to dig deeply into research. She put together a list of priority projects, some initiated at Imperial, others discussed at AIM, as well as new ideas using algebraic topology that had been developed at Oxford.

The freedom of being on such a long fellowship has enabled Heather to take full advantage of the talented community at Oxford as well as to interact with the wider research community. She is currently supervising PhD students with other members of Oxford’s math-bio group and is interacting with faculty from different areas of mathematics including algebraic topology and numerical analysis. During the time of the fellowship, she has acquired funding to hire two postdocs to work with her. Along with Adam Maclean (now at UC Irvine), she collaborated with mathematicians at UC Berkeley to use algebraic matroids to study chemical reactions and experimental design, specifically applied to molecular interactions that malfunction in colon cancer. With Emilie Dufresne (now at the University of Nottingham, and an expert in invariant theory and commutative algebra), she explored the “geometry of sloppiness,” which studies uncertainty in parameter estimation. This work, in turn, has led to new mathematical questions that are rooted in algebraic geometry and moduli space problems. Heather and Emilie are currently exploring how invariant theory can help with model identification problems.

Heather’s research, which has its roots in our department, is coming full circle back to UMass. Indeed, Emilie and Heather have just started a collaboration with Panos Kevrekidis to use techniques in computer algebra to study vortex problems. One of Heather’s long-term research goals is to combine techniques from computational algebraic topology to perform model comparison, specifically on the same type of models investigated in her UMass REU on tumor-induced angiogenesis models with simulated or experimental data. The advances in computational topology and recent availability of spatio-temporal vascular network data make now the perfect time to develop such mathematical methods, which ultimately could be applied more widely to other scientific problems.

Colleagues at Oxford have also provided a supportive environment for Heather, including guiding her through a successful application for a Royal Society University Research Fellowship, which she started in January 2017. On a more personal note, Mariano and Heather are now both in Oxford’s Mathematical Institute, and they are expecting their first child in May 2017. They are looking forward to the adventure of starting a family and balancing this with their active academic careers.

Heather has become active in outreach activities in the UK, and has fond memories of our department, especially of Nate and Panos, who served as her role models. Her teaching style is motivated by their inspirational approach in the classroom, and she still seeks their advice for decisions on research and career decisions.
The following alumni and friends have made generous contributions to the Department of Mathematics and Statistics between January 2016 and June 2017. Your gifts help us improve our programs and enrich the educational experiences of our students. We deeply appreciate your continuing support.

INDIVIDUAL DONORS
($1000+)
John and Patricia Baillieul
Joan and Edgar Barksdale, Jr.
Donald Catlin
Peter and Anne Costa
Jonathan Fienup
Christine Fraser
Robert and Louise Fredette
Barbara Hamkalo
Keith Hartt and Ann Wiedie
Youping Huang
Alvin Kho
Annah Lincoln
John Loughlin
Steven and Geni Monahan
Robert and Veronica Piziak
Robert and Lynne Pollack
Stuart Rachlin
Peng Wang and Yuanyuan Chen

INDIVIDUAL DONORS
($250-999)
Anonymous
David and Joan Bodendorf
Linda Bonavia
Jonathan Camire
Carol and William Cox, Jr.
Elizabeth Doane
John Ehrhardt
Dominic Herard
Gerald Janowitz
Matthew Kindzerske
Hsu-Tung and Mei-Chin Ku
Marilyn Lacerte
Rachel and Donald Levy
Chunming Li
Chandler Lincoln III
Carolyn MacArthur
Dwight Manley
Shirley Merriam
Barry Randall
Brian Riley
Cathleen Riley
Gerard Sarnie
Jonathan Skinner
Robert and Cindy Tardiff
Fusheng Wei

INDIVIDUAL DONORS
Anonymous
Geraldine Amprimo
Samuel Antiles
Henri and Susan Azibert
Robert Babeau
June Bacon
Carl and Joanne Balduf
Robert and Debra Bashford
Richard Bates
Laura Beltis
Philip Blau
David Bookston
Daniel Bourdeau
Margaret and Carl Britton, Jr.
Mary Francis Brzezenski
Richard Burns
Ronald Burt
Yuan Cao
James and Carol Chanen
Yuyu Chen
Donna Chevaire
Frances Chevarley
Michael and Kristen Cincotti
Richard Coco
Diane and Kevin Coghlan
Paul Connolly
Mary Ann Connors
D. John Coparanis
Leonard and Leona Dalton
John Davey
John Demoy
Che Der
Kyle DeSormier
Neil Doherty
Mei Duanmu
Constance Duffy
Patricia Dysart
Emily Dzwil
Adam and Jill Eisner
Neil Falby
Kenneth Frail
Emily Fritzman
Adam Gamzon
Joshua Gay and Meredith Beaton
Mary Ann Godbout
Linda Goldberg
William Goodhue, Jr.
Roy and Patricia Greenwood
James Griffin, Jr.
John and Pamela Guertin
Jeremy Guertin
Douglas and Rose Mary Haddad
Heather Harrington
Donald Hastings
David Hennessy
Joseph and Melanie Higgins
Neil and Audrey Hindman
Renee Janow
Thomas and Marianne Kalmbach
John Kelley
Richard Kennedy
John Ketler
Mizan Khan
Leo Korn
Claudia Kulesh
Elizabeth Kumm
Vicki Kuziak
Anna Kye
Lorraine Lavallee
Marguerite Lawton
Mark and Evelyn Leeper
It's easy to donate to the Department of Mathematics and Statistics. To make an online gift, please visit the Department's donations page at www.math.umass.edu/donate.

If you prefer to donate by check, please make it payable to "UMass Amherst Mathematics" and mail to:

Christine Curtis
Grant and Budget Administrator
Department of Mathematics and Statistics
Lederle Graduate Research Tower
710 North Pleasant Street
University of Massachusetts
Amherst, MA, 01003-9305

To learn more about supporting the Department of Mathematics and Statistics, please contact Christine Curtis at curtis@math.umass.edu or 413-545-0095.
KUDOS TO THE NEW SENIOR VICE PROVOST FOR ACADEMIC AFFAIRS!

Professor Farshid Hajir has been named Senior Vice Provost for Academic Affairs by acting Provost and Senior Vice Chancellor for Academic Affairs John McCarthy, effective 1 October 2017. Hajir fills the position vacated by McCarthy when he became acting Provost in July.

Since 2006, Farshid has served our department in various administrative roles, first as undergraduate program director, then as associate head, and for the past three years as department head. During this time, he led initiatives to establish a calculus help center; to revamp the mathematics major curriculum; to revise and improve academic and career advising for our majors, whose numbers have doubled over the past four years; and to increase the diversity of our faculty.

Hajir led our department through a strategic planning process, with subsequent hiring that has strengthened our research groups in applied probability, statistics, analysis, and geometry, and that has begun to build a research group in discrete mathematics. He also successfully advocated to establish the Marshall H. Stone Visiting Assistant Professorship and expand the scope of our VAP program.

Farshid’s development efforts in the department have helped to secure funding for over 80 students to participate in research experiences for undergraduates over the past decade, as well as to create and endow the new John Balbiileul Distinguished Lectures (a series of 3 talks by an eminent scholar each year). He has also served the university through membership on the Campus Planning and Resource Committee, the General Education Council, the Faculty Senate, and as the Faculty Senate’s delegate to the Board of Trustees.

“Hajir is an ideal choice,” said McCarthy. “He is a first-rate researcher who also has a broad knowledge of undergraduate and graduate education, who has spearheaded important initiatives in his department, and who has worked across department and college boundaries.”