ADVANCED EXAMS

ALGEBRA

I. Group Theory and Representation Theory

Group actions; counting with groups. p-groups and Sylow theorems. Composition series; Jordan-Holder theorem; solvable groups. Automorphisms; semi-direct products. Structure of finitely generated Abelian groups

II. Representations of groups.

Complex representations of finite groups: Schur's Lemma; Maschke's Theorem; representations of Abelian groups; Characters; Schur's orthogonality relations.

II. Linear Algebra and Commutative Algebra.

Euclidean domain implies PID implies UFD. Gauss Lemma; Eisenstein's Criterion. Exact sequences; isomorphism theorems for modules. Free modules. Hom and tensor product of vector spaces, Abelian groups, and modules; Restriction and extension of scalars.

Bilinear forms; symmetric and alternating forms; symmetric and exterior algebras. Structure Theorem for finitely generated modules of a PID. Rational canonical form. Jordan canonical form.

Chain conditions; Noetherian rings and modules; Hilbert's Basis Theorem. Prime and maximal ideals. Field of fractions. Localization of rings and modules: exactness of localization; local rings; Nakayama's Lemma. Integral extensions. Noether's Normalization Lemma. Integral closure. Nullstellensatz. Closed affine algebraic sets.

III. Field Theory and Galois Theory

Algebraic field extensions: finite extensions; degree of extensions; the minimal polynomial; adjoining roots of polynomials; the splitting field; algebraic closure. Separable extensions. Theorem of the primitive element. Galois extensions: Fundamental Theorem of Galois Theory; Finite fields and their Galois groups. Frobenius endomorphism. Cyclotomic polynomial. Cyclotomic fields and their Galois groups. Cyclic extensions. Solvable extensions. Solving polynomial equations in radicals. Transcendence degree.

REFERENCES

Dummit and Foote, *Abstract Algebra*

Atiyah and MacDonald, *Introduction to Commutative Algebra*

Serre, *Linear Representations of Finite Groups*

Lang, *Algebra*

ANALYSIS

- **Lebesgue measure**: Construction of Lebesgue measure on $\mathbf{R}$ and $\mathbf{R^d}$. Measurable and non measurable sets; Cantor sets. Lebesgue and Borel measurable functions; Egorov theorem and Lusin Theorems. Construction and properties of Lebesgue integral; the space L^1 of integrable functions and its completeness; comparison with Riemann integral. Fubini-Tonelli theorem in $\mathbf{R^d}$. Modes of convergence: convergence almost everywhere, convergence in measure, convergence in L^1.

- **Integration and differentiation**: Differentiation of the integral; Hardy-Littlewood maximal function; Lebesgue differentiation theorem. Functions of bounded variation; absolutely continuous functions; the fundamental theorem of calculus.

- **Hilbert spaces** Abstract Hilbert spaces and examples; L^2 spaces; Bessel's inequality and Parseval's identity; Riemann-Lebesgue Lemma; Orthogonality; orthogonal projections. Linear transformations; linear functionals; Riesz representation theorem; adjoints transformations.

- **Fourier analysis**: Fourier transform in L^1 and L^2; Fourier inversion formula. Fourier series; Dirichlet's Theorem and Fejér's Theorem

- **General theory of measure and integration** Measure spaces and σ-algebras. σ-finite measures. Caratheodory theorem and the construction of measures; outer measures and extension theorems. Integration theory. Product measures and Fubini-Tonelli theorem. Signed measure; Radon-Nikodym theorem; Borel measures on $\mathbf{R}$ and Lebesgue-Stieljes integral.

- **Banach spaces and L^p spaces**. Abstract Banach spaces and examples; completeness criterion. Convexity; L^p-norms; Schwarz, Hölder, Minkowski, and Jensen inequalities. L^p-spaces and their duals; Riesz-Fischer Theorem.

REFERENCES:

Berberian, Introduction to Hilbert Spaces
Folland, Real Analysis
Gelbaum and Olmsted, Counterexamples in Analysis
Halmos, Measure Theory
Royden, Real Analysis

Rudin, Principles of Mathematical Analysis
Rudin, Real and Complex Analysis
Stein and Shakarchi, Real Analysis
Wheeden and Zygmund, Measure and Integral

DIFFERENTIAL EQUATIONS

- Constant coefficient linear systems of ODE, normal forms, exponential matrix solutions, variation of parameters formula.
- Well-posedness of the initial-value problem: local existence, uniqueness and continuous dependence for nonlinear systems of ODE; continuation and global existence; Picard's (iteration) method and Euler's (finite difference) method; Gronwall's inequality.
- Limit sets and invariant sets: equilibria, limit cycles, ω and α-limit sets; invariant manifolds (stable, unstable, center).
- Stability theory: linearization at an equilibrium point, Lyapunov stability.
- Two-dimensional systems (plane autonomous systems): phase portraits, Poincaré–Bendixson theory; qualitative analysis of special systems, such as gradient or Hamiltonian systems.
- Elementary facts about distributions: weak derivatives and mollifiers, convolutions and the Fourier transform.
- The prototype linear equations of hyperbolic, elliptic and parabolic type: the wave, potential (Laplace/Poisson), and diffusion (heat) equations; fundamental solutions for these equations.
- Initial-value problems for PDE: the hyperbolic Cauchy problem (wave equation), basic features of wave propagation (D'Alembert's solution and Kirchhoff's solution), characteristics, energy estimates; the parabolic Cauchy problem (diffusion equation), basic features of diffusion phenomena.
- Boundary-value problems for elliptic PDE: the Laplace and Poisson equations with Dirichlet, Neumann or periodic boundary conditions; variational formulation and weak solutions, the Sobolev spaces H^1 and H^1_0; the associated eigenvalue problem and eigenfunction expansions; Green's functions; the maximum principle.
- Mixed initial/boundary-value problems: separation of variables and eigenfunction expansion methods.

REFERENCES

M. Hirsch and S. Smale, *Differential Equations, Dynamical Systems, and Linear Algebra*

L. Perko, *Differential Equations and Dynamical Systems*

J. Hale, *Ordinary Differential Equations*

P. Garabedian, *Partial Differential Equations*

F. John, *Partial Differential Equations*

I. Stakgold, *Boundary Value Problems of Mathematical Physics, I and II*

F. Treves, *Basic Linear Partial Differential Equations*

V.S. Vladimirov, *Equations of Mathematical Physics*

GEOMETRY

- Inverse and implicit function theorems, rank of a map. Regular and critical values. Sard's theorem.
- Differentiable manifolds, submanifolds, embeddings, immersions, submersions, diffeomorphisms.
- Tangent space and bundle, differential of a map. Partitions of unity, orientation, transversality, embeddings in $\mathbb{R}^n$.
- Vector fields, local flows, Lie bracket, Frobenius theorem.
- Lie groups (generalities), matrix Lie groups, left-invariant vector fields, Lie algebra of a Lie group.
- Tensor algebra, tensor fields, differential forms, the exterior differential, integration, Stokes theorem, closed and exact forms, deRham's cohomology.
- Vector bundles, normal bundle. Operations on vector bundles.
- Surfaces in $\mathbb{R}^3$. Gaussian and mean curvature. The Theorema Egregium.
- Rudiments of Riemannian geometry. The Riemannian (Levi–Civita) connection. The curvature tensor.

REFERENCES

Auslander and Mackenzie, *Introduction to Differentiable Manifolds*

Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*

do Carmo, *Differential Geometry of Curves and Surfaces*

Guillemin and Pollack, *Differential Topology*

Morgan, *Riemannian Geometry*

Spivak, *Differential Geometry, Vol. I, II*

Warner, *Foundations of Differentiable Manifolds and Lie Groups*

ADVANCED STATISTICS VERSION I

Content of basic probability and statistics and random vectors, multivariate distribution, the multivariate normal, linear and quadratic forms including non-central t, chi-square and F distributions; basic theory for inferences (estimation, confidence intervals, hypothesis testing, power) for the general linear model including both full rank regression and correlation models as well as some treatment of less than full rank models arising in the analysis of variance

REFERENCES

References for basic Statistics and Probability exams

Ravishanker and Dey, *A First Course in Linear Model Theory.* Chapters 1, 2, 3, 5 and 6.

Graybill, *Theory and Application of the Linear Model.* Chapters 3 and 4.

Seber and Lee, *Linear Regression Analysis 2nd Edition.* Chapters 1 and 2.

ADVANCED STATISTICS VERSION II

Content of basic probability and statistics and statistical models; point estimation, set estimation and hypothesis testing from a frequentist's, decision theoretic and Bayesian point of view; finite sample and asymptotic techniques in a variety of (parametric/semiparametric/non-parametric) statistical models.

REFERENCES

References for basic Statistics and Probability exams

Jun Shao, *Mathematical Statistics (2nd Edition).* Springer Series in Statistics.

T. S. Ferguson, *A Course in Large Sample Theory.* Chapman & Hall/CRC

A. W. van der Vaart, *Asymptotic Statistics.* Cambridge Series in Statistical and Probabilistic Mathematics

LITERATURE-BASED ORAL EXAM

1. **Inference in Graphical Models and Bayesian Networks**

   Primary faculty: Patrick Flaherty

   Readings:

   - Bishop C. "Pattern Recognition and Machine Learning". Chapters 8-14

2. **Statistical Inference and Convex Optimization**

   Primary faculty: Patrick Flaherty

   Readings:

   - Boyd S and Vandenbergh L. "Convex Optimization" Chapters 2-7.
   - StanfordX CVX101 https://lagunita.stanford.edu/courses/Engineering/CVX101/Winter2014/about

3. **Causal Modeling**

   Primary faculty: Krista Gile

   Readings:

   - Pearl, "Causal inference in statistics: An overview", Statistics Surveys, Vol. 3, 96–146, 2009.
   - Rosenbaum and Rubin, "The Central Role of the Propensity Score in Observational Studies for Causal Effects, Biometrika, Vol 70 No 1, 41-55, 1983
   - "Smoking and lung cancer: recent evidence and a discussion of some questions" , Cornfield et al, 1959, reprinted in International Journal of Epidemiology 2009;38 :11751191

4. **Mixture/latent class model for heterogeneous data**

   Primary faculty: Daeyoung Kim

   Readings:

   - Finite Mixture and Markov Switching Models (Sylvia Frhwirth-Schnatter) : Chapter 1, 2, 3 (3.1, 3.2, 3.3, 3.5, 3.7), 4
   - Medical Applications of Finite Mixture Models (Peter Schlattmann) : Chapter 2 (2.1) and Chapter 4

5. **Copulas for modeling multivariate dependence**

   Primary faculty: Daeyoung Kim

   Readings:

   - Financial Engineering with copulas explained (Jan-Frederik Mai and Matthias Scherer) : Chapter 1-4, 6, 7
   - A paper concerning Copula-Based Regression (a primary faculty will give a relevant paper to a student after a student chooses this topic)

6. **Splines for regression and density estimation**

   Primary faculty: Anna Liu

   Readings:

   - Introduction to statistical learning with Applications in R by Gareth James, Daniela Witten, Trevor Hastie and Robert Tibshirani June 2014. Chapter 7: 7.1-7.5, 7.7, 7.8
   - Elements of statistical learning by Trevor Hastie, Robert Tibshirani, Jerome H. Friedman February 2009. Chapter 5: 5.2, 5.4, 5.5, 5.8
   - Smoothing Spline ANOVA Models by Chong Gu, 2013. Chapter 7: 7.1, 7.3-7.5

7. **Markov chain Monte Carlo for Bayesian estimation and inference**

   Primary faculty: John Staudenmayer

   Readings:

   - Bayesian Computation with R. (Albert, chapter 6)
   - Bayesian Data Analysis (Gelman, Carlin, Stern, Rubin, Part 3)
   - The Bayesian Choice (Robert, chapter 6)
   - Handbook of Markov Chain Monte Carlo (Brooks, Gelman, Jones, and Meng)