DEPARTMENT OF MATHEMATICS AND STATISTICS
UMASS - AMHERST
BASIC EXAM - STATISTICS
WINTER 2010

1. Suppose you want to determine the fraction of cars that pass by a certain corner during the morning commute that are 25 years old or older. Call that fraction p. (Also, suppose you can look at a car and instantly determine its age.) You decide to collect two datasets to address this question. In experiment (A) you watch 100 cars and record the number that are 25 years old or older. Call that number x. In experiment (B) you watch the cars until you see one car that is 25 years old or older, and you record the number of cars you saw (including the old one). Call that number y.

   (a) What is the likelihood for p based on experiment (A)?

   (b) What is the likelihood for p based on experiment (B)?

   (c) What is the maximum likelihood estimator for p based on experiment (A)? (You must derive the MLE and show that it's a MLE.)

   (d) What is the maximum likelihood estimator for p based on experiment (B)? (You must derive the MLE and show that it's a MLE.)

   (e) Suppose you only had time to do one of these experiments. Which one would you choose and on what statistical criteria did you base your answer?

2. Let $Y_1, \ldots, Y_n$ be a simple random sample of size n from a Poisson distribution with *pmf* $f(x; \lambda) = \exp(-\lambda)\lambda^x/x!, x = 0, 1, 2, \ldots$. Let $\theta = Pr(Y > 0)$.

   (a) Find the MLE of θ and show that it is consistent.

   (b) Is the estimator you found in (a) unbiased?

   (c) What is the asymptotic distribution of $\sqrt{n}(\hat{\theta} - \theta)$ where $\hat{\theta}$ is the MLE?

   (d) Suppose we want to find an unbiased estimator for θ.

      i. Let $S = \sum_{i=1}^{n} y_i$. You may use that $S \sim$ Poisson$(n\lambda)$. With K as a constant, what is the expectation of K^S?

      ii. Use the pervious result to get an unbiased estimator for θ.

3. Let $X_1, \ldots, X_n$ be a random sample from a uniform distribution on 0 to θ.

   (a) Write down the likelihood function for θ.

   (b) Find a one dimensional sufficient statistic for θ.

   (c) Find the MLE for θ. (You must derive the MLE and show that it's a MLE.)

   (d) The Pareto(a, b) distribution has *pdf* $f(y; a, b) = \frac{ab^a}{y^{a+1}}, y \geq b$ (and 0 otherwise). If a prior for θ is pareto(a, b), then what is the posterior?

   (e) Use your answer to (d) to find an estimator for θ.

(f) Based on your answer to (d), describe how you could find an interval estimator for θ.

(g) Describe how your choice of b influences the posterior.