Department of Mathematics and Statistics
University of Massachusetts

Basic Exam: Topology

August 31, 2001

Answer five of seven questions. Indicate clearly which five questions you want to have graded. Justify your answers.

Passing Standard: For Master's level, 60% with two questions essentially complete. For Ph.D. level, 75% with three questions essentially complete.

1. Consider these topologies on $\mathbb{R}$:

   (i) trivial (= indiscrete topology)

   (ii) Zariski (= finite complement topology)

   (iii) standard (= order topology)

   For each topology, determine which of these functions $\mathbb{R} \to \mathbb{R}$ is continuous (with the same topology on both copies of $\mathbb{R}$):

   $$(a)\ \sin x,\ (b)\ x^3,\ (c)\ e^x$$

2. Let A be a subspace of $\mathbb{R}$ (standard topology) with a finite number of connected components. Prove that its complement $B = \mathbb{R} \setminus A$ also has a finite number of components. If A has countably many components, is it true that B has countably many components (give a proof or a counterexample)?

3. Give an example of a topological space which is connected but not locally connected.

4. Let X be the closed unit disk in $\mathbb{R}^2$. Show that for any space Y, projection

   $$q : X \times Y \to Y$$

   is a closed map.

5. Let $f : X \to X$ be a continuous map from a compact Hausdorff space X to itself. Verify that its set of fixed points

   $$F = \{x \in X | f(x) = x\}$$

   is also compact.

6. Prove (without quoting big theorems) that a compact metric space is complete.

7. Let F be the set of continuous functions from $[0, 1]$ to itself.

   (a) Show that the topology of pointwise convergence is not the same as the compact-open topology on F.

   (b) Show that F is not compact in the compact-open topology.