

POWER SERIES/MATH 513

- (1) Suppose d divides n . Use the identity

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

to verify the identity

$$\frac{1}{1-x^d} = \sum_{i=0}^{n/d-1} \frac{x^i}{1-x^n}.$$

- (2) The *Bell numbers* b_n appear in the generating function

$$e^{e^x-1} = \sum_{n \geq 0} \frac{b_n}{n!} x^n.$$

Use this expression to check that the first four Bell numbers are 1, 1, 2, 5.

- (3) Define a sequence by

$$a_0 = 1, a_1 = 1, a_k = 2a_{k-1} + a_{k-2}, \quad k \geq 2.$$

Express the generating function $f(x) = \sum a_n x^n$ explicitly as a rational function $P(x)/Q(x)$ and find an explicit formula for a_n in terms of the roots of $Q(x)$.