CAYLEY GRAPHS/MATH 513

(1) Is it possible for a Cayley graph to be a complete graph? Either explain why it is not possible, or give for any n a pair of a group G and a generating set S such that the corresponding Cayley graph is K_n.

(2) Compute the Cayley graph for $G = SL_2(\mathbf{Z}/3\mathbf{Z})$, where S consists of the matrices

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

(3) Compute the Cayley graph for $G = S_4$ (symmetric group on 4 letters), where $S = \{(1,2),(2,3),(3,4)\}$. (Hint: the Cayley graph for S_3 given in class will appear a few times as a subgraph.)

(4) Given two groups G, H, the *product group* $G \times H$ consists of all pairs $(g,h), g \in G, h \in H$ with multiplication given componentwise: $(g,h) \cdot (g',h') = (gg', hh')$. If S_G generates G and S_H generates H, then the set of pairs $S_G \times S_H$ will generate $G \times H$.

   (a) Draw the Cayley graph of $(\mathbf{Z}/3\mathbf{Z}) \times (\mathbf{Z}/4\mathbf{Z})$, where the generating set is taken to be $(1,1), (-1,-1)$.

   (b) Explain what happens for $(\mathbf{Z}/m\mathbf{Z}) \times (\mathbf{Z}/n\mathbf{Z})$, where again the generating set is taken to be $(1,1), (-1,-1)$.